A Long-Term Behavioral New Keynesian Model

Christopher D. Cotton
Columbia University *

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Abstract

The Behavioral New Keynesian model appears to offer a simple way to resolve many paradoxes within Macroeconomics. I demonstrate, however, that a key feature of the model is that the irrational part of agents' expectations are fixed at the steady state level. I relax this assumption and allow for the irrational part of expectations to update slowly in line with what agents observe. I derive an alternative Behavioral New Keynesian model within this long-term expectations framework. In this case, a fixed nominal interest rate rule is not determinate and the zero lower bound does not have bounded costs. Thus, important paradoxes remain unresolved. This contrasts with Gabaix (2017b).

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1 Introduction

In recent years, economists have introduced behavioral features into standard models to help to resolve apparent paradoxes - one such example is the Behavioral New Keynesian Model. To help explain features that rational expectations cannot explain, many economists have investigated introducing non-rational features into standard rational models. One popular example is Gabaix (2017b) which introduces a Behavioral New Keynesian model. The assumptions of this model are that agents are only able to perceive some fraction of correct expectations.

The Behavioral New Keynesian model helps to resolve many apparent paradoxes within standard models. We are particularly interested in two of these paradoxes. Firstly, the Behavioral New Keynesian shows that a fixed nominal interest rate is determinate. The fact that this is not determinate in usual models would imply that a country which remains at the zero lower bound for a long period, such as Japan, would experience significant volatility which does not appear to be realistic. Secondly, the Behavioral New Keynesian shows that remaining at the zero lower bound for a long period of time has a bounded cost unlike a standard model where remaining at the zero lower bound has increasingly large costs. The former case appears more reasonable because there did not appear to be increasingly large costs of remaining at the zero lower bound during the recent financial crisis or for Japan generally.

My main contribution is to demonstrate that once we allow for long-term updating to the behavioral expectations, the aforementioned paradoxes are not resolved. A key assumption behind the Behavioral New Keynesian model is that part of the expectations of an agent are imperfect and determined only by the steady state, which does not update even after long persistent deviations away from steady state. I allow for this imperfect part of expectations to instead be determined by long-run expectations which update very slowly. I derive a New Keynesian Model which I argue better fits a discussion of long-run issues. I show within this framework that fixed interest rates are indeterminate and that the zero lower bound has unboundedly negative costs, like in the standard New Keynesian model but in contrast to the Behavioral New Keynesian model.

It appears more realistic to model the imperfect component of expectations as based upon long-term expectations rather than the steady state. There are two problems with modeling that agents base the imperfect part of their expectations on the steady state. The first is that if there are persistent deviations away from steady state in one direction then agents' expectations will be biased towards the steady state. Of course, if we think agents do place some weight on the steady state even under persistent biased deviations then this is still reasonable. However, the second reason why this is problematic is that it is unclear that agents do have a clear conceptual understanding of what steady state is. Even with inflation where there is a clear target in place, Kumar et al. (2015) demonstrate that agents have a wide and biased conception of the likely future value of inflation. And the key variable here for which agents need a clear conceptual idea of steady state is actually consumption for which its not clear what is the value in steady state.

Introducing long-term expectations which slowly update means that there is only effectively a discounted Euler equation in the short-term but not the long-term. Once we allow for the imperfect part of agents' expectations to update slowly towards realised values then under persistent repeated shocks, the discounting disappears in the long-term. The reason a Behavioral New Keynesian model can generate determinacy of a fixed rate rule is because the only way the output gap can rise today is if it rises explosively in the future since agents only increase their consumption today if they expect higher consumption in the future and their expectations are biased downwards due to discounting. When we remove the long-term discounting, this result no longer holds since agents effectively don't discount in the long-term. Similarly, the zero lower bound produces bounded costs in the Behavioral New Keynesian model since the output gap in the current period doesn't fully respond to negative output gaps in the future. When we remove the long-term discounting this result again disappears.

A related set of papers that this paper clearly relates to is Gabaix's framework of sparse dynamic programming. Gabaix (2014) introduced the basic idea of sparse dynamic programming. Gabaix (2017a) discussed how this could be applied broadly to Macroeconomics. Gabaix (2017b) applied this to the New Keynesian model to generate the Behavioral New Keynesian model.

My paper also relates to a wider literature studying the impact of introducing behavioral features into a New Keynesian model. One such paper is Woodford (2018). Woodford considers a different behavioral framework where agents have finite horizons with some long-run value function. If you allow the long-run value function to update in response to agent's observations then this can lead to similar responses to the long-run framework I propose based upon sparse dynamic programming. There are some differences such as Woodford finds that a fixed interest rate peg yields a unique (but unstable) equilibrium whereas I find a fixed interest rate is non-unique.

More broadly, this paper relates to recent papers on the discounted Euler equation. Typically, we expect that if consumers raise their consumption in the future by 1p.p. then they will consumption smooth and raise their consumption by 1p.p. in the present ceteris paribus. Euler discounting allows for the possibility that consumers don't raise their consumption today as much in response to future consumption. Gabaix (2017b) introduces the discount by effectively fixing a portion of the future expectations of consumption. McKay et al. (2017) also introduce a discounted Euler equation model. The discounting there comes from the fact that in the future with some probability agents will receive a fixed amount of consumption. Broadly, a discounted Euler equation relies upon fixing some component of future consumption or expectations of future consumption to be constant. This makes a lot of sense in the short-run or when things change on a one-off basis like in forward guidance. However, in the long-term, this is not an innocuous assumption.

In section 2, I review the differences between the method Gabaix uses to solve for long-term expectations and the method I use in the context of a simple consumption allocation problem. In section 3, I summarise the key results of interest in Gabaix (2017b). In section 4, I derive my long-term Behavioral New Keynesian model and compare its properties to Gabaix's model.

2 Simple Example

In this section, we consider how sparse dynamic programming affects a simple example. We demonstrate that the implications do not appear reasonable in the long-term. We propose an alternative form of sparse dynamic programming which yields the same results as sparse dynamic programming but does not suffer from the flaws of sparse dynamic programming in the long-term.

Setup We are going to consider the themes of the paper within a simple consumption allocation problem. An agent lives for infinite periods. Each period they consume C_t which yields utility $u(C_t)$. They maximise their lifetime utility and have a discount rate β. Agents receive income Y_t each period. We rewrite income in log-linearised terms i.e. $Y_t = \bar{Y}\exp(\hat{Y}_t)$ where $\hat{Y}_t = \log(Y_t) - \log(\bar{Y})$ and $\bar{Y}$ is steady state income. Any income that agents do not consume is saved for the next period and is denoted S_{t+1}. Agents start each period with savings they made in the last period S_t on which they have received a fixed return of $1+r$. We allow for behavioral expectations, denoted $\mathbb{E}^b$. Each period, they solve the following problem to compute their consumption today C_t and the amount they save for the next period S_{t+1}:

$$\max_{\{S_{t+1+i}, C_{t+i}\}_{i=0}^{\infty}} \mathbb{E}_t^b[\sum_{i=0}^{\infty} \beta^i u(C_{t+i})]$$

s.t.

$$C_{t+i} + S_{t+i+1} = \bar{Y}\exp(\hat{Y}_{t+i}) + (1+r)S_{t+i}$$

Standard Sparse Dynamic Programming Solution The basic principle of sparse dynamic programming is that agents have imperfect expectations of variables that are realized in the future. In earlier work, Gabaix derives the information costs that can generate these imperfect expectations. However, in later work, Gabaix mainly takes these processes as given and assumes specific imperfect expectations. We will follow his example and take as given agent's expectations of future income shocks.

We set that the agent only learns some fraction M of shocks. To keep the formulae simple, we assume this is true even of shocks in the current period but all the results would go through without this. Formulaically, we are assuming that:

$$\mathbb{E}_t[\hat{Y}_{t+i}] = M\mathbb{E}_t[\hat{Y}_{t+i}]\forall i \geq 0$$

where $\mathbb{E}$ represents the rational expectations beliefs about the shock. Notice that when $M = 1$, we are in the rational expectations case. $M < 1$ represents the behavioral case where agents learn about the future imperfectly.

To make things even simpler, we assume that future shocks are fully learnable in advance with enough information processing. Therefore, $\mathbb{E}_t[\hat{Y}_{t+i}] = \hat{Y}_{t+i}$. We can then rewrite the agent's problem at time t without expectations:

$$\max_{\{S_{t+i+1},C_{t+i}\}_{i=0}^{\infty}} \sum_{i=0}^{\infty} \beta^i u(C_{t+i})$$

s.t.

$$C_{t+i} + S_{t+i+1} = \bar{Y}\exp(M\hat{Y}_{t+i}) + (1+r)S_{t+i} \tag{1}$$

We can sum the single period budget constraints (equation 8) to get the lifetime budget constraint:

$$\sum_{i=0}^{\infty} \frac{C_{t+i}}{(1+r)^i} = \sum_{i=0}^{\infty} \frac{\bar{Y}\exp(M\hat{Y}_{t+i})}{(1+r)^i} + (1+r)S_t \tag{2}$$

By taking first order conditions, we derive our usual Euler condition relating consumption in the present with the future $u'(C_t) = \beta(1+r)u'(C_{t+1})$. We set r so that $\beta(1+r) = 1$. Therefore, the consumer saves with the anticipation that $C_t = C_{t+1}$ and we can therefore simplify the lifetime budget constraint (equation 2) to get an expression for consumption at time t:

$$C_t = \frac{r}{1+r}\sum_{i=0}^{\infty} \frac{\bar{Y}\exp(M\hat{Y}_{t+i})}{(1+r)^i} + rS_t \tag{3}$$

We consider the case where there is a persistent deviation in the consumer's income away from steady state. We set that $Y_t = \exp(a)\bar{Y}$. We assume that $a > 0$ which implies that $Y_t > \bar{Y}$ indefinitely. We can rewrite this as $\log(Y_t) = a + \log(\bar{Y})$ or $\hat{Y}_t = a$. This allows us to simplify the solution for consumption a lot:

$$C_t = \bar{Y}\exp(Ma) + rS_t \tag{4}$$

We can also rewrite the single period budget constraint (equation 8) as the change in savings:

$$S_{t+1} - S_t = Y_t - C_t + rS_t \tag{5}$$

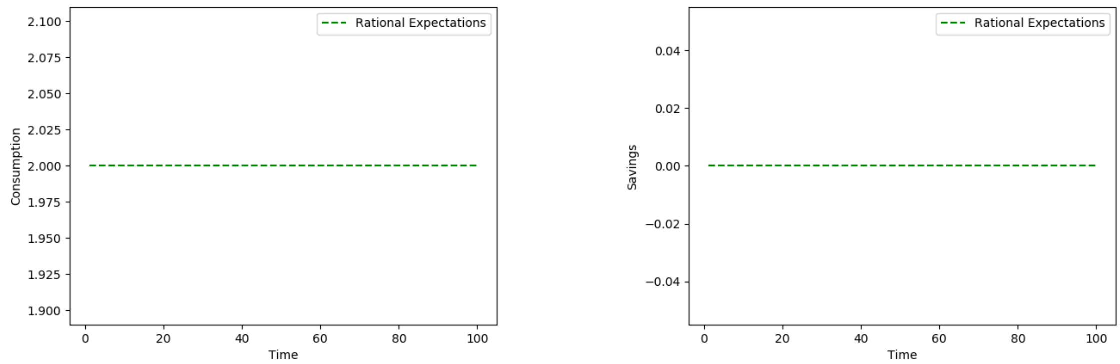
Inputting the fact that $Y_t = \bar{Y}\exp(a)$ and our solution for consumption (equation 4) into the change in savings equation (equation 5) we just derived yields:

$$S_{t+1} - S_t = \bar{Y}[\exp(a) - \exp(Ma)] \tag{6}$$

Rational Expectations Path With rational expectations, $M = 1$ because the agent fully anticipates future shocks. We plot this in figure 1. We observe that consumption and savings remain the same. We can also see this from our equation for the change in savings (equation 6).

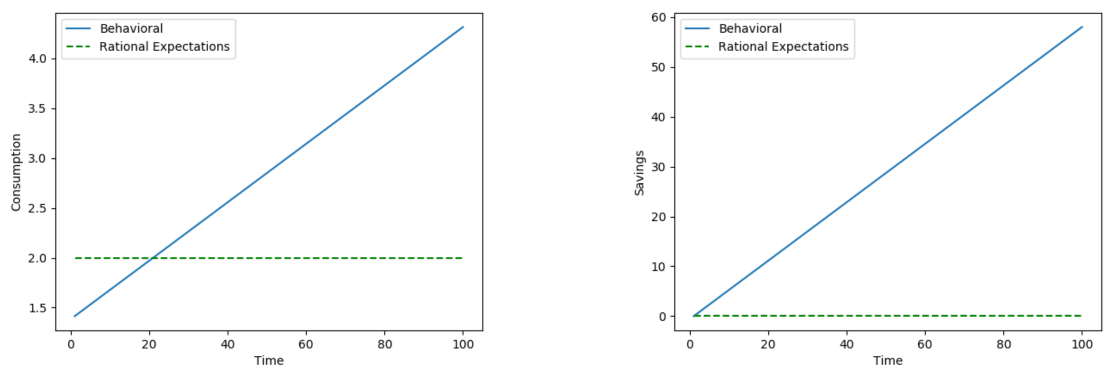
This makes sense because we satisfy conditions for the permanent income hypothesis to hold so the agent should consume the same over time. And since their income remains the same, they should just consume their income over time.

Figure 1: Rational Expectations



(a) Consumption (b) Savings

Figure 2: Standard Sparse Dynamic Programming



(a) Consumption (b) Savings

Standard Sparse Dynamic Programming Path Now, let's set $M < 1$ to capture behavioral features where the agent doesn't fully anticipate future shocks. We show the path of consumption and savings in figure 2. We observe that consumption and savings rise indefinitely in contrast to the rational expectations case. We can also see this from equation 6 where savings rises by the same constant amount every period: $\bar{Y}(\exp(a) - M\exp(a))$.

The intuition here is that the agent again tries to apply the permanent income hypothesis and consume the same every period. However, they underestimate income in exactly the same way forever. They anticipate that they will have an income each period of $\bar{Y}\exp(Ma)$ when actually each period their income will be $\bar{Y}\exp(a)$.

The reason we get such a different result to the rational expectations case is that with sparse dynamic programming (when $M < 1$) we persistently underestimate income in the same way forever.

We can see why the agent persistently underestimates income by noting that the steady state level of log deviations in income from steady state will just be zero ($\bar{\hat{Y}} = 0$). Then we get:

$$\mathbb{E}_t^b[\hat{Y}_{t+i}] = M\mathbb{E}_t[\hat{Y}_{t+i}] + (1-M)\bar{\hat{Y}} \tag{7}$$

We observe that sparse dynamic programming is effectively setting that expectations of a variable are determined by the weighted sum of actual expectations and the value of that variable in steady state. So when there are persistent deviations in a particular direction away from steady state, the behavioral expectations will be biased towards the steady state. In our case, income is persistently higher than steady state so the agent underestimates income and therefore consumes too little and saves too much indefinitely.

Long-Term Sparse Dynamic Programming Solution To combat this problem, we introduce a form of sparse dynamic programming that does not fix some component of the expectations to forever be the steady state level.

To do this, we adjust equation 7 so that the incorrect part of expectations are based upon long-term expectations of a variable as opposed to based upon the steady state. We set long-term expectations $\mu_{\hat{Y},t}$ to be some slowly updating variable reflecting the agent's observations over time. We set $\alpha_{\hat{Y}}$ to be the extent to which long-term expectations of $\hat{Y}_t$) are updated. Thus, behavioral expectations under alternative sparse dynamic programming are given by:

$$\mathbb{E}_t^b[\hat{Y}_{t+i}] = M\mathbb{E}_t[\hat{Y}_{t+i}] + (1-M)\mu_{\hat{Y},t}$$

where:

$$\mu_{\hat{Y},t} = \alpha_{\hat{Y}}\hat{Y}_t + (1-\alpha_{\hat{Y}})\mu_{\hat{Y},t-1}$$

Thus, the agent's problem is slightly different to the sparse dynamic programming case due to the long-term expectations term:

$$\max_{\{S_{t+i+1},C_{t+i}\}_{i=0}^{\infty}} \sum_{i=0}^{\infty} \beta^i u(C_{t+i})$$

s.t.

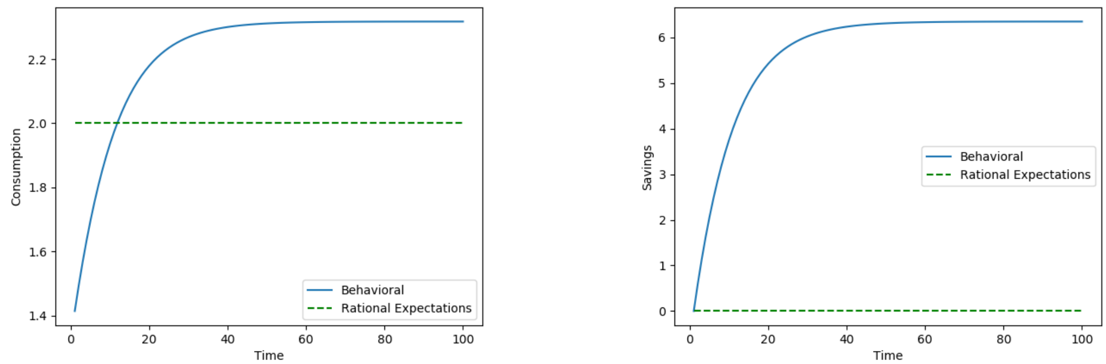
$$C_{t+i} + S_{t+i+1} = \bar{Y}\exp(M\hat{Y}_{t+i} + (1-M)\mu_{\hat{Y},t}) + (1+r)S_{t+i} \tag{8}$$

We can then follow the same simplifying steps to get to a very similar solution for consumption as equation 4. We rewrite the single period budget constraint (equation 8) as a lifetime budget constraint and apply the Euler condition to get:

$$C_t = \bar{Y}\exp(M\log(A) + (1-M)\mu_{\hat{Y},t}) + rS_t \tag{9}$$

In the same way as before, we can input the fact that $Y_t = \bar{Y}\exp(a)$ and our solution for

Figure 3: Long-Term Sparse Dynamic Programming



(a) Consumption

(b) Savings

consumption (equation 9) into the change in savings equation (equation 5) to yield:

$$S_{t+1} - S_t = \bar{Y}[\exp(a) - \exp(Ma + (1-M)\mu_{\hat{Y},t})] \tag{10}$$

Long-Term Sparse Dynamic Programming Path We assume that the agent has only received steady state income so far. Therefore, their long-term expectations will be to receive the steady state level of income so they expect the deviation of income from steady state will be zero i.e. $\mu_{\hat{Y},0} = 0$. The path of consumption and savings under long-term sparse dynamic programming is given in figure 3.

Initially, the consumer believes that future income will equal $\exp(Ma)$, like in the sparse dynamic programming case. Consequently, initially, the agent saves a positive amount, like in normal sparse dynamic programming, because they underestimate the value of future income.

However, over time, the long-term expectations of $\hat{Y}$ update to take into account the persistent deviation in $\hat{Y}$. In the long-term they will converge to the updated value of $\hat{Y} = a$. Thus, $\lim_{t\to\infty} \mu_{\hat{Y},t} = a$. Therefore, over time, the amount we save will converge to 0 like in the rational expectations case.

3 Model Setup

Here we describe the basic setup of the New Keynesian model with behavioral features. This setup generates both Gabaix's Behavioral New Keynesian model and my alternative Behavioral New Keynesian model. We can also get the standard 3 equation New Keynesian model from this setup.

3.1 Households

There is an infinitely lived agent. Their utility is improved by consuming more and working less. Each period they can consume C_t or buy nominal bonds B_{t+1}. They receive a wage W_t for the amount they work L_t and a nominal return i_t on their nominal bonds from the previous period B_t.

$$\max \mathbb{E}_t^b[\sum_{i=0}^{\infty} \beta^i [u(C_{t+i}) - v(L_{t+i})]]$$

s.t.

$$P_t C_t + B_{t+1} = P_t W_t L_t + P_t profits_t + B_t(1 + i_t)$$

Taking first order conditions:

$$u'(C_t) = \mathbb{E}_t[\beta \frac{1 + i_t}{1 + \pi_{t+1}} u'(C_{t+1})]$$

$$W_t u'(C_t) = v'(L_t) \tag{11}$$

We define the real interest rate R_t to equal the real return offered by holding bonds in equation 12.

$$1 + r_t = \mathbb{E}_t[\frac{1 + i_t}{1 + \pi_{t+1}}] \tag{12}$$

3.2 Firms

We use the standard firm setup with monopolistic intermediate goods firms producing with linear labor-only productions and facing price rigidities, and a CES competitive firm aggregating the intermediate goods.

This yields (under Calvo pricing rigidities with an inflation target of zero or Rotemberg price rigidities with any inflation target):

$$\pi_t = \kappa_2 MC_t + \beta \mathbb{E}_t[\pi_{t+1}] \tag{13}$$

We could have introduced behavioral agents on the firm side like in Gabaix's behavioral New Keynesian model but all this really does is lower the β parameter. It doesn't qualitatively change the resulting model.

3.3 Monetary Policy

Unless specified otherwise, we assume the central bank pursues a simple Taylor Rule:

$$i_t = \phi_\pi \pi_t \tag{14}$$

3.4 Equilibrium

$$Y_t = C_t = W_t L_t + profits_t \tag{15}$$

Note that even if there are no future shocks the path that agents decide at time t may not be what they do at $t+1$ because we allow for the possibility of non-rational expectations

4 Standard Behavioral New Keynesian Model and its Applications

In this section, I provide a summary of sparse dynamic programming (Gabaix, 2017a), the Behavioral New Keynesian model and the long-term results that Gabaix derives on Behavioral New Keynesian models Gabaix (2017b).

4.1 Standard Sparse Dynamic Programming

Here I discuss briefly the form of the behavioral expectations which is outlined broadly in Gabaix (2017a).

The key assumption of sparse dynamic programming agents pay only limited attention to current/future exogenous variables. We set that $M_{Z,t+i,t}$ is a vector of the attention paid to the exogenous variables Z_{t+i} when the agent is at time t. To keep things simple, we assume that Z_{t+i} have a zero steady state value. Then really all we are doing is assuming that behavioral expectation of Z_{t+i} are only some fraction of the true rational expectations. We can express this in equation 16.

$$\mathbb{E}_t^b[Z_{t+i}] = \mathbb{E}_t[Z_{t+i,t}] = M_{Z,t+i,t}\mathbb{E}_t[Z_{t+i}] \tag{16}$$

We observe that the agent only partly bases their expectations of exogenous variable Z_{t+i} on the true expectations. The degree to which they do so is based upon $M_{Z,t+i,t}$. When $M_{Z,t+i,t} = 1 \forall t, i$ we are in the rational expectations case. Gabaix (2017a) provides a method for generating appropriate values of $M_{Z,t+i,t}$ based upon the concept of sparsity. However, he points out that typically we can just take the value of $M_{Z,t+i,t}$ as given and proceed without considering its importance. This is the case for the Behavioral New Keynesian model, which we are particularly interested in.

4.2 Standard Behavioral New Keynesian Expectations

For the behavioral New keynesian model, Gabaix assumes that the degree of attention to the future is such that households with behavioral expectations only receive some portion $\bar{m}^i$ of the correct signal about a state i periods ahead. Therefore, behavioral expectations are equivalent to equation 17

$$\mathbb{E}_t^b[\hat{\mathbb{X}}_{t+k}] = \bar{m}^k \mathbb{E}_t[\hat{\mathbb{X}}_{t+k}] \tag{17}$$

where $\hat{\mathbb{X}}_{t+k}$ are the log-linearised states at time $t+k$.

For example, if $m = 0.5$ and a rational agent would be able to decipher that their log-linearised income will rise by 0.01 in 2 periods, a behavioral agent would only observe that it will rise by 0.0025.

4.3 Standard Behavioral IS Curve

We define the natural level of consumption i.e. realised consumption without price ridigities, to be C_t^n. We denote log-linearised consumption with a hat. We then derive our usual measure for the output gap (equation 18) and the log-linearised real rate (equation 19)

We can define x_t, r_t to be respectively the log-linear deviations of C_t, R_t. We define x_t to be the level of log-linearised consumption relative to the natural case (denoted $\hat{C}_t^n$)

$$\hat{x}_t = \hat{C}_t - \hat{C}_t^n \tag{18}$$

$$\hat{r}_t = \log(R_t) - \log(\bar{R}) \tag{19}$$

We show in appendix B.1 that in equilibrium we get an IS curve given by equation 39.

$$\hat{x}_t = M\mathbb{E}_t[\hat{x}_{t+1}] - \sigma(\hat{i}_t - \mathbb{E}_t[\hat{\pi}_{t+1}] - \hat{r}_t^n) \tag{20}$$

where $0 < M < 1$.

The derivation is given in appendix B.1. Intuitively, it makes sense that we discount the future by M relative to the rational expectations case since x_{t+1} is a function of states at time $t+1$ which are all discounted by M relative to the equivalent states at t.

To keep things simple, we assume that $\hat{r}_t^n = 0$ always.

4.4 Standard Behavioral New Keynesian Model

We already have our IS curve (equation 39).

We can combine equations 11, 13 and 15 in the usual way to get our standard Phillips curve (equation 21).

$$\hat{\pi}_t = \kappa\hat{x}_t + \beta\mathbb{E}_t[\hat{\pi}_{t+1}] \tag{21}$$

We also know that the central bank pursues a simple Taylor rule (equation 14). We then get a system of 3 equations (equations 14, 21 and 39) and 3 unknowns (x_t, π_t, i_t).

We see that the key qualitative difference between the behavioral New Keynesian model and the standard New Keynesian model is the M in equation 39. This is a form of Euler discounting since agents react less today to future deviations in output from steady state than they would in the standard case (where $M = 1$).

4.5 Application 1: Determinacy of Fixed Interest Rate Rule

When we have a fixed rate rule then we always have that i_t does not deviate from its steady state level $\bar{i}$ and thus $\hat{i}_t = 0$. We can simplify equation 39 to equation 22 as a result.

$$\hat{x}_t = \bar{m}\mathbb{E}_t[\hat{x}_{t+1}] + \sigma\mathbb{E}_t[\hat{\pi}_{t+1}] \tag{22}$$

Gabaix demonstrates that there are parameter values for which this simple Behavioral New Keynesian model will be determinate for a fixed interest rate rule. This contrasts with the standard New Keynesian model where the model is always indeterminate for a fixed interest rate rule.

To explore the intuition for why we can get determinacy through the Behavioral New Keynesian model, let's consider a simplified model. We set $\beta = 0$ in the Phillips Curve. All this means is that that firms don't take into account price rigidities when they set their prices. equation 21 then simplifies to yield $\hat{\pi}_t = \kappa\hat{x}_t$. We can input this into equation 22 to get equation 23

$$\hat{x}_t = (\bar{m} + \sigma\kappa)\mathbb{E}_t[\hat{x}_{t+1}] \tag{23}$$

We see that there is a relationship between output now and in the future. This relationship comes primarily from the Euler equation. Agents want to spread their consumption over time rather than having high consumption in one period and then low consumption in the next. An additional effect is that the only way in which output can be higher in the future is if price rise in the future (since firms will raise prices in response to high output). A rise in prices implies a lower real rate and thus agents want to consume even more in the present. This is why a 1p.p. rise in output in the present requires a $<$ 1p.p. rise in output in the future under rational expectations i.e. $\bar{m} = 1$.

The rational expectations case is always indeterminate. Indeterminacy implies that there are multiple non-explosive expected paths that can be followed by variables in a model. In other words, we can pick any value for $\hat{x}_t$ without the model exploding. We see this is possible here under rational expectations since if $\hat{x}_t$ rises by 1p.p. then output in the future will rise by less and we will slowly return to the steady state (as opposed to exploding).

A rise of 1p.p. in output today can require a rise of more than 1p.p. in the future in the Behavioral New Keynesian model. Behavioral New Keynesian agents still conduct consumption smoothing. So, for agents to consume 1p.p. more today they still need to believe that their consumption will rise in the future. Since agents have limited attention to the future, they underestimate the degree to which output will rise in the future. Therefore, there needs to be a larger increase in future output than in the standard New Keynesian model to generate a 1p.p. increase in output today. If the degree of inattention is large enough then it can be that a 1p.p. rise in output requires a more than 1p.p. rise in output in the future.

The Behavioral New Keynesian model can be determinate. If agents do not pay that much attention to the future then $\bar{m}$ can be low enough that $\bar{m} + \sigma\kappa < 1$. In this case, we get the opposite of the rational expectations case. The only way it can make sense to raise output by 1p.p. in the current period is if output would rise by more than 1p.p. in the future. However, this would require that over time, output would have to keep growing to infinity. Thus, any path where $\hat{x}_t \neq 0$ is explosive and we therefore have determinacy since there is only one path $\hat{x}_t = 0$ which is not explosive.

4.6 Application 2: Costs of Zero Lower Bound

We investigate the costs of being at the zero lower bound for a long period of time.

We consider an experiment where the natural real rate of interest stays indefinitely at some constant level low enough to force the economy to remain at the zero lower bound. When $r^n < -\pi^\star$, it is necessary to set $i < 0$ to achieve stable inflation. This is not thought to be possible due to the zero lower bound. So we consider an experiment where $r^n < -\pi^\star$ indefinitely.

We discuss how to compute this experiment in appendix B.2. We can show that the ccosts of hitting the ZLB persistently can be bounded.

Under rational expectations, the costs must intuitively be unbounded. A 1p.p. fall in $\mathbb{E}_t[\hat{x}_{t+1}]$ implies a 1p.p. fall in $\hat{x}_t$. Therefore, if the real interest rises due to the central bank's inability to lower the nominal interest rate, the output gap at t must be persistently non-neglibly lower than at $t+1$. We know this will continue indefinitely and therefore the costs of remaining at the zero lower bound are unbounded.

With Gabaix's behavioral expectations, the costs can intuitively be bounded. A 1p.p. fall in $\mathbb{E}_t[\hat{x}_{t+1}]$ implies a Mp.p. fall in $\hat{x}_t$ where $M < 1$. Therefore, even if the real interest rate is positive, agents do not react fully to the future expected fall in the output gap. Therefore, the output gap can converge to some bounded level given in appendix B.2.

5 Long-Term Behavioral New Keynesian Model and its Applications

5.1 Long-Term Sparse Dynamic Programming

One issue with the sparse dynamic programming approach in the long-term is that sparse dynamic programming requires that part of expectations are fixed. As the examples in **??** showed, this can mean that even after income or the real interest rate has been constant at a value away from steady state indefinitely, expectations about the future will be consistently biased towards the fixed expectations and the consumer will never learn.

A second issue with the sparse dynamic programming approach in the long-term is that it requires that agents know steady state values for variables in equilibrium. This is reasonable for some variables like shocks (where the steady state will just be zero) or inflation (where the steady state is likely to conform to a target). However, for most variables, for example income, it seems likely that agent's do not have a clear idea of what is steady state. A more realistic assumption might be to assume that agents have some long-term views about the likely values of variables in steady state but that these views can change over time.

I propose instead that agent's beliefs about a variable Z_{t+i} have some component of weight $M_{Z,t+i,t}$ that is correctly based upon rational expectations (like in standard sparse dynamic programming) but also a second component of weight $1-M_{Z,t+i,t}$ that is based upon long-term beliefs about Z_t which slowly update as new information about Z_t is learnt. Therefore, we set:

$$\mathbb{E}_t^b[Z_{t+i}] = M_{Z,t+i,t}\mathbb{E}_t[Z_{t+i}] + (1 - M_{Z,t+i,t})\mu_{Z,t} \tag{24}$$

$\mu_{Z,t}$ are the slowly updating non-rational beliefs of the agent about Z at time t. We typically assume that:

$$\mu_{Z,t+1} = \chi_Z Z_t + (1 - \chi_Z)\mu_{Z,t}$$

where:

$$0 < \chi_Z < 1$$

So each period the consumer updates their beliefs about Z_t based upon what happened in the last period and their past beliefs. Raising χ_Z makes the agent update their beliefs more quickly.

This framework will remove the need for agents to have clear knowledge about a steady state value of $\bar{Z}$ and can allow for the consumer to gradually improve their understanding of persistent deviations from steady state.

5.2 Long-Term Behavioral New Keynesian Expectations

Expectations in the long-term Behavioral New Keynesian model are given in equation 25. They are an application of equation 24 to the case where agents only learn some proportion $\bar{m}^i$ of future variables that occur i periods in the future in the short-term.

$$\mathbb{E}_t^b[\hat{\mathbb{X}}_{t+i}] = \bar{m}^i\mathbb{E}_t[\hat{\mathbb{X}}_{t+i}] + (1 - \bar{m}^i)\mu_{\hat{X},t} \tag{25}$$

where $\hat{\mathbb{X}}_{t+k}$ are the log-linearised states at time $t+k$.

$$\mu_{\hat{X},t+1} = \chi_{\hat{X}}\hat{X}_t + (1 - \chi_{\hat{X}})\mu_{\hat{X},t}$$

5.3 Long-Term Behavioral IS Curve

We again define $\hat{x}_t, \hat{r}_t$ to be respectively the output gap and the log-linearised real interest rate. Applying section 5.1 to section 3.1 yields:

$$x_t = -\sigma r_t + M\mathbb{E}_t[x_{t+1}] + (1 - M)\mu_{x,t} + D_x(\mu_{x,t} - \mathbb{E}_t[\mu_{x,t+1}]) \tag{26}$$

$$\mu_{x,t+1} = \chi_x x_t + (1 - \chi_x)\mu_{x,t} \tag{27}$$

where $0 < M < 1$.

The derivation is given in appendix C.1.

5.4 Long-Term Behavioral New Keynesian Model

We already have our IS curve (equations 26 and 27).

We still have the same standard New Keynesian Phillips Curve (equation 21) and monetary rule (equation 14). Therefore, we have four equations (equations 14, 21, 26 and 27) and four unknowns (x_t, r_t, i_t, π_t).

5.5 Application 1: Determinacy of Fixed Interest Rate Rule

The long-term behavioral New Keynesian model is only determinate when $\phi_\pi > 1$. This contrasts with the standard New Keynesian model where the model can be determinate even under a fixed interest rate rule i.e. $\phi_\pi = 0$ (section 4.5). We demonstrate this result in appendix C.2.

It is important to stress that this holds for an arbitrarily small updating of long-term expectations i.e. arbitrarily small χ_x. It could take hundreds of years for long-term expectations to update but still the model will only be determinate if nominal interest rates respond by more than one-to-one to inflation.

The intuition here is that in the long-term, we move back to the case where a 1p.p. rise in output today does not necessitate an explosive rise in future output. The reason for this is that if income rises today, agents will adapt their expectations gradually to this change and therefore there will need to be a relatively smaller rise in future periods to justify why the consumer chooses to consume 1p.p. more today. Therefore, consuming 1p.p. more today does not generate an explosive path unless we adopt a standard Taylor rule like in the standard (non-behavioral) New Keynesian model.

5.6 Application 2: Costs of the Zero Lower Bound

In the long-term Behavioral New Keynesian model, the costs of the zero lower bound are unbounded. We demonstrate this in appendix C.3.

The output gap cannot converge to some stable level that bounds the costs of the zero lower bound, unlike in the standard Behavioral New Keynesian model. In the short-term, the output gap may converge to some relatively stable level since agents don't respond fully now to output gaps in the future due to imperfect observation of the future. However, we cannot converge to this output gap completely because in the long-term the expectations of agents about income will fall so the output gap will fall, and this process will continue indefinitely.

6 Conclusion

I relax the assumption in the Behavioral New Keynesian model that part of expectations always depend upon the steady state. I instead consider a framework in which the imperfect part of expectations within the behavioral framework slowly update according to agents' observations. Within this setup, some of the key results of the Behavioral New Keynesian model reverse. It is no longer possible for a less than one-to-one response of nominal interest rates to inflation (including a fixed nominal interest rate rule) to generate determinacy. This is the same as the New Keynesian model but the opposite of the Behavioral New Keynesian model. And the zero lower bound produces unbounded increasing costs. Again, this is the same as the New Keynesian model but the opposite of the Behavioral New Keynesian model.

Appendices

A Model Setup Details

A.1 Euler Condition

We need to linearise the Euler condition (**??**) and the intertemporal budget constraint (**??**). Let's start with the Euler condition. We can express the Euler condition as a general function:

$$f(C_t, C_{t+1}, R_{t+1}) = C_t^{-\gamma} - \beta\mathbb{E}_t[R_{t+1}C_{t+1}^{-\gamma}]$$

We can then just apply a standard Taylor first-order approximation around C_t, C_{t+1}, R_{t+1}:

$$\bar{C}_t^{-\gamma} - \beta\bar{R}\bar{C}_t^{-\gamma} - \gamma\bar{C}_t^{-\gamma-1}(C_t - \bar{C}_t) + \gamma\beta\bar{R}\bar{C}_t^{-\gamma-1}(\mathbb{E}_t[C_{t+1}] - \bar{C}_t) - \beta\bar{C}_t^{-\gamma}(\mathbb{E}_t[R_{t+1}] - \bar{R})$$

Multiplying by $\bar{C}_t^{-\gamma-1}$ and noticing that the first part cancels:

$$-\gamma(C_t - \bar{C}_t) + \gamma\beta\bar{R}(\mathbb{E}_t[C_{t+1}] - \bar{C}_t) - \beta\bar{C}_t(\mathbb{E}_t[R_{t+1}] - \bar{R})$$

$$\mathbb{E}_t[C_{t+1}] - \bar{C}_t = (C_t - \bar{C}_t) + \frac{1}{\gamma}\frac{\bar{C}_t}{\bar{R}}(\mathbb{E}_t[R_{t+1}] - \bar{R}) \tag{28}$$

We can iterate equation 28 to yield:

$$\mathbb{E}_t[C_{t+i}] = C_t + \frac{1}{\gamma}\frac{\bar{C}_t}{\bar{R}}\sum_{k=1}^{i}(\mathbb{E}_t[\hat{R}_{t+k} - \mathbb{R}) \tag{29}$$

A.2 Budget Constraint

We define:

$$W_t = \frac{B_t}{P_{t-1}}(1 + r_{t-1})$$

The individual budget constraints become:

$$W_{t+i+1} = R_{t+i+1}(W_{t+i} - C_{t+i} + Y_{t+i})$$

(Note that this is different to Gabaix who sets: $W_{t+i+1} = R_{t+i+1}(W_{t+i} - C_{t+i}) + Y_{t+i}$. Gabaix

(2017a, p.13))

Intertemporal budget constraint in this case is:

$$\sum_{i=0}^{\infty} \frac{1}{\Pi_{j=1}^{i} R_{t+j}} C_{t+i} = W_t + \sum_{i=0}^{\infty} \frac{1}{\Pi_{j=1}^{i} R_{t+j}} Y_{t+i}$$

We see that:

$$\frac{1}{1-\frac{1}{\bar{R}}}\bar{C}_t = W_t + \frac{1}{1-\frac{1}{\bar{R}}}\bar{Y}$$

$$\bar{C}_t = W_t(1-\frac{1}{\bar{R}}) + \bar{Y}$$

$$\bar{C}_t = W_t \frac{\bar{R}-1}{\bar{R}} + \bar{Y}$$

The Euler condition linearisation remains the same.

We define the following function for the linearisation of the intertemporal budget constraint:

$$f(C_t, C_{t+1}, \ldots, Y_t, Y_{t+1}, \ldots, R_t, R_{t+1}, \ldots) = \sum_{i=0}^{T} \frac{C_{t+i}}{\Pi_{j=1}^{i} R_{t+j}} - \sum_{i=0}^{T} \frac{Y_{t+i}}{\Pi_{j=1}^{i} R_{t+j}} - W_t$$

We get the following Taylor approximation of **??**:

$$\approx 0 + \sum_{i=0}^{T} \frac{1}{\bar{R}^i}(C_{t+i} - \bar{C}_t) - \sum_{i=0}^{T} \frac{1}{\bar{R}^i}(Y_{t+i} - \bar{Y}) - \sum_{i=1}^{T}\sum_{k=i}^{T} \frac{\bar{C}_t - \bar{Y}}{\bar{R}^{k+1}}(R_{t+i} - \bar{R})$$

$$\sum_{i=0}^{T} \frac{1}{\bar{R}^i}(C_{t+i} - \bar{C}_t) - \sum_{i=0}^{T} \frac{1}{\bar{R}^i}(Y_{t+i} - \bar{Y}) - \sum_{i=1}^{T}\sum_{k=i}^{T} \frac{\bar{C}_t - \bar{Y}}{\bar{R}^{k+1}}(R_{t+i} - \bar{R}) = 0 \tag{30}$$

We just focus upon the latter real interest rate term. We also set $T = \infty$.

$$\sum_{i=1}^{\infty}\sum_{k=i}^{\infty} \frac{\bar{C}_t - \bar{Y}}{\bar{R}^{k+1}}(R_{t+i} - \bar{R})$$

$$= \sum_{i=1}^{\infty} \frac{\bar{R}-1}{\bar{R}} W_t (R_{t+i} - \bar{R}) \bar{R}^{-i-1} \frac{1}{1-\frac{1}{\bar{R}}}$$

$$= \sum_{i=1}^{\infty} \frac{\bar{R}-1}{\bar{R}} W_t (R_{t+i} - \bar{R}) \bar{R}^{-i-1} \frac{\bar{R}}{\bar{R}-1}$$

$$= \sum_{i=1}^{\infty} W_t (R_{t+i} - \bar{R}) \bar{R}^{-i-1}$$

$$= W_t \sum_{i=0}^{\infty} \bar{R}^{-i-1}(R_{t+i} - \bar{R})$$

A.3 Solution

We can take expectations, input the Euler condition and simplify to get:

$$\frac{1}{1-\frac{1}{\bar{R}}}(C_t - \bar{C}_t) = \sum_{i=0}^{\infty} \bar{R}^{-i}(\mathbb{E}_t[Y_{t+i}] - \bar{Y}) + W_t \sum_{i=1}^{\infty} \bar{R}^{-i-1}(\mathbb{E}_t[R_{t+i}] - \bar{R}) - \frac{1}{\bar{R}-1}\frac{1}{\gamma}\bar{C}_t \sum_{i=1}^{\infty} \bar{R}^{-i}(\mathbb{E}_t[R_{t+i}] - \bar{R})$$

Rewriting:

$$C_t - \bar{C}_t = \frac{\bar{R}-1}{\bar{R}} \sum_{i=0}^{\infty} \bar{R}^{-i}(\mathbb{E}_t[Y_{t+i}] - \bar{Y}) + \frac{\bar{R}-1}{\bar{R}} W_t \sum_{i=1}^{\infty} \bar{R}^{-i-1}(\mathbb{E}_t[R_{t+i}] - \bar{R}) - \frac{1}{\gamma}\bar{C}_t \sum_{i=1}^{\infty} \bar{R}^{-i-1}(\mathbb{E}_t[R_{t+i}] - \bar{R})$$

Or:

$$C_t - \bar{C}_t = \mathbb{E}_t^b[\sum_{i=0}^{\infty} \bar{R}^{-i}[b_y(Y_{t+i} - \bar{Y}) + b_r(W_t)(R_{t+i} - \bar{R})]] \tag{31}$$

$$b_y = \frac{\bar{R}-1}{\bar{R}}$$

$$b_r(W_t) = \frac{\bar{R}-1}{\bar{R}^2} W_t - \frac{1}{\gamma}\bar{C}_t \frac{1}{\bar{R}}$$

Using the definition of $\bar{C}_t$, we can also get:

$$b_r(W_t) = (1 - \frac{1}{\gamma})\frac{\bar{R}-1}{\bar{R}^2} W_t - \frac{1}{\gamma}\frac{1}{\bar{R}}\bar{Y}$$

B Standard Behavioral New Keynesian Model Details

B.1 Standard Behavioral New Keynesian IS Curve

Applying behavioral expectations to equation 31

$$C_t - \bar{C}_t = \mathbb{E}_t[\sum_{i=0}^{\infty} \bar{R}^{-i}[M_{Y,i} b_y \hat{Y}_{t+i} + M_{R,i} b_r(W_t)\hat{R}_{t+i}]] \tag{32}$$

We follow Gabaix (2017b, Section 2.1) and assume that:

$$M_{Y,i} = \bar{m}^i m_y$$

$$M_{R,i} = \bar{m}^i m_r$$

We allow for a slightly more general case than the one discussed in the main text by allowing for additional inattention through m_y, m_r.

Inputting this into equation 32 yields:

$$C_t - \bar{C}_t = \mathbb{E}_t[\sum_{i=0}^{\infty}(\frac{\bar{m}}{\bar{R}})^i[m_y b_y \hat{Y}_{t+i} + m_r b_r(W_t)\hat{R}_{t+i}]] \tag{33}$$

We can also make the assumption that $W_t = 0$ which would occur if there was no exogenous savings mechanism. Then:

$$b_r(W_t) = -\frac{1}{\gamma}\frac{1}{\bar{R}}\bar{Y}$$

We also note that in this case:

$$\bar{C}_t = \bar{C} = \bar{Y}$$

Thus, we can simplify equation 33 to:

$$C_t - \bar{C} = \mathbb{E}_t[\sum_{i=0}^{\infty}(\frac{\bar{m}}{\bar{R}})^i[m_y b_y \hat{Y}_{t+i} - m_r \frac{1}{\gamma}\frac{1}{\bar{R}}\bar{Y}\hat{R}_{t+i}]] \tag{34}$$

Next, we define:

$$x_t = \frac{C_t - \bar{C}}{\bar{C}}$$

$$r_t = \frac{R_t - \bar{R}}{\bar{R}}$$

Inputting this into equation 34 yields:

$$x_t = \mathbb{E}_t[\sum_{i=0}^{\infty}(\frac{\bar{m}}{\bar{R}})^i[m_y b_y x_{t+i} - \frac{1}{\gamma}m_r r_{t+i}]] \tag{35}$$

We can simplify this as follows:

$$x_t = m_y b_y x_t - \frac{1}{\gamma}m_r r_t + \mathbb{E}_t[\sum_{i=1}^{\infty}(\frac{\bar{m}}{\bar{R}})^i[m_y b_y x_{t+i} - \frac{1}{\gamma}m_r r_{t+i}]]$$

$$x_t = m_y b_y x_t - \frac{1}{\gamma}m_r r_t + \frac{\bar{m}}{\bar{R}}\mathbb{E}_t[\sum_{i=0}^{\infty}(\frac{\bar{m}}{\bar{R}})^i[m_y b_y x_{t+1+i} - \frac{1}{\gamma}m_r r_{t+1+i}]]$$

$$(1 - m_y b_y)x_t = -\frac{1}{\gamma}m_r r_t + \frac{\bar{m}}{\bar{R}}\mathbb{E}_t[x_{t+1}]$$

$$x_t = \frac{\bar{m}}{\bar{R}(1 - m_y b_y)}\mathbb{E}_t[x_{t+1}] - \frac{1}{\gamma}\frac{m_r}{1 - m_y b_y}r_t$$

Therefore, we get:

$$x_t = M\mathbb{E}_t[x_{t+1}] - \sigma r_t \tag{36}$$

where:

$$M = \frac{\bar{m}}{\bar{R}(1 - m_y b_y)}$$

$$= \frac{\bar{m}}{\bar{R}(1 - m_y \frac{\bar{R}-1}{\bar{R}})}$$

$$= \frac{\bar{m}}{\bar{R} - m_y(\bar{R} - 1)} \tag{37}$$

$$\sigma = \frac{1}{\gamma} \frac{m_r}{1 - m_y b_y} \tag{38}$$

$$= \frac{1}{\gamma} \frac{m_r}{1 - m_y \frac{\bar{R}-1}{\bar{R}}}$$

B.2 Costs of Zero Lower Bound in Standard Behavioral New Keynesian Model

Call r_z the low level of r that the economy gets stuck at indefinitely.

$$r_z \leq -\pi^\star$$

Then, denoting $\hat{i}_z$ to be the value that $\hat{i}_z$ takes at the zero lower bound:

$$\hat{r}_z \leq -\pi^\star - r^n = -\bar{i} = \hat{i}_z$$

We set $\hat{r}_z - \hat{i}_z = -0.01$. We can then behavioral IS curve as:

$$\hat{x}_t = M\mathbb{E}_t[\hat{x}_{t+1}] - \sigma(0.01 - \mathbb{E}_t[\hat{\pi}_{t+1}]) \tag{39}$$

The NKPC stays the same.

We also note that when the ZLB does end and we return to normal then if we follow a simple Taylor rule i.e. $i_t = \phi_\pi \pi_t$ then we would need to have $\pi_t = 0$ at the conclusion of the ZLB and this would require $x_t = 0$. We can then just work backwards from this final period to simulate what will happen. This is shown on Gabaix (2017b, p.22).

Another way of observing this is to observe that $\hat{x}$ can converge to a steady state level. The steady state of the IS and NKPC curves is respectively:

$$\bar{\hat{x}} = M\bar{\hat{x}} - \sigma(0.01 - \bar{\hat{\pi}})$$

$$\bar{\hat{\pi}} = \kappa\bar{\hat{x}} + \beta\bar{\hat{\pi}}$$

Combining these:

$$\bar{\hat{x}} = M\bar{\hat{x}} - \sigma(0.01 - \frac{\kappa}{1-\beta}\bar{\hat{x}})$$

Rearranging:

$$\left[1 - M - \frac{\kappa}{1-\beta}\right]\bar{\hat{x}} = -\sigma(0.01)$$

As long as the term in square brackets is positive, $\bar{\hat{x}}$ will converge.

C Long-Term Behavioral New Keynesian Model Details

C.1 Long-Term Behavioral New Keynesian IS Curve

We follow Gabaix (2017b, Section 2.1) and assume that:

$$M_{Y,i} = \bar{m}^i m_y$$

$$M_{R,i} = \bar{m}^i m_r$$

We can simplify **??** to get:

$$C_t - \bar{C}_t = \mathbb{E}_t[\sum_{i=0}^{\infty}(\frac{\bar{m}}{\bar{R}})^i[m_y b_y \hat{Y}_{t+i} + m_r b_r(W_t)\hat{R}_{t+i}]] + \sum_{i=0}^{\infty}\bar{R}^{-i}[b_y(1-\bar{m}^i m_y)\mu_{\hat{Y},t}]$$

$$C_t - \bar{C}_t = \mathbb{E}_t[\sum_{i=0}^{\infty}(\frac{\bar{m}}{\bar{R}})^i[m_y b_y \hat{Y}_{t+i} + m_r b_r(W_t)\hat{R}_{t+i}]] + [\frac{1}{1-\frac{1}{\bar{R}}} - \frac{m_y}{1-\frac{\bar{m}}{\bar{R}}}]b_y\mu_{\hat{Y},t} \tag{40}$$

We can also make the assumption that $W_t = 0$ which would occur if there was no exogenous savings mechanism. Then **??** becomes:

$$b_r(W_t) = -\frac{1}{\gamma}\frac{1}{\bar{R}}\bar{Y}$$

We also note that in this case:

$$\bar{C}_t = \bar{C} = \bar{Y}$$

Thus, we can simplify equation 40 to get:

$$C_t - \bar{C} = \mathbb{E}_t[\sum_{i=0}^{\infty}(\frac{\bar{m}}{\bar{R}})^i[m_y\frac{\bar{R}-1}{\bar{R}}\hat{Y}_{t+i} - m_r\frac{1}{\gamma}\frac{1}{\bar{R}}\bar{Y}\hat{R}_{t+i}]] + [\frac{1}{1-\frac{1}{\bar{R}}} - \frac{m_y}{1-\frac{\bar{m}}{\bar{R}}}]\frac{\bar{R}-1}{\bar{R}}\mu_{\hat{Y},t}$$

$$C_t - \bar{C} = \mathbb{E}_t[\sum_{i=0}^{\infty}(\frac{\bar{m}}{\bar{R}})^i[m_y\frac{\bar{R}-1}{\bar{R}}\hat{Y}_{t+i} - m_r\frac{1}{\gamma}\frac{1}{\bar{R}}\bar{Y}\hat{R}_{t+i}]] + [1 - \frac{m_y(\bar{R}-1)}{\bar{R}-\bar{m}}]\mu_{\hat{Y},t} \tag{41}$$

Next, we define:

$$x_t = \frac{C_t - \bar{C}}{\bar{C}}$$

$$r_t = \frac{R_t - \bar{R}}{\bar{R}}$$

$$\mu_{x,t} = \frac{\mu_{\hat{Y},t}}{\bar{Y}}$$

We note that we can Y_t in **??** with $\mu_{x,t}$ to get:

$$\mu_{x,t+1} = \chi_y x_t + (1-\chi_y)\mu_{x,t} \tag{42}$$

We can then simplify equation 41 to get:

$$x_t = \mathbb{E}_t[\sum_{i=0}^{\infty}(\frac{\bar{m}}{\bar{R}})^i[m_y b_y x_{t+i} - \frac{1}{\gamma}m_r r_{t+i}]] + [1 - \frac{m_y(\bar{R}-1)}{\bar{R}-\bar{m}}]\mu_{x,t} \tag{43}$$

Now, we can rewrite equation 43 with $\mathbb{E}_t[x_{t+1}]$:

$$x_t = m_y b_y x_t - \frac{1}{\gamma}m_r r_t + \frac{\bar{m}}{\bar{R}}\mathbb{E}_t[\sum_{i=0}^{\infty}(\frac{\bar{m}}{\bar{R}})^i[m_y b_y x_{t+1+i} - \frac{1}{\gamma}m_r r_{t+1+i}]] + \frac{\bar{m}}{\bar{R}}[[1 - \frac{m_y(\bar{R}-1)}{\bar{R}-\bar{m}}](\mathbb{E}_t[\mu_{x,t+1}] - \mathbb{E}_t[\mu_{x,t+1}]) + [1 - \frac{m_y(\bar{R}-1)}{\bar{R}-\bar{m}}]\mu_{x,t}$$

$$x_t(1 - m_y b_y) = -\frac{1}{\gamma}m_r r_t + \frac{\bar{m}}{\bar{R}}\mathbb{E}_t[x_{t+1}] + \frac{\bar{m}}{\bar{R}}[[1 - \frac{m_y(\bar{R}-1)}{\bar{R}-\bar{m}}](\mu_{x,t} - \mathbb{E}_t[\mu_{x,t+1}]) + (1 - \frac{\bar{m}}{\bar{R}})[1 - \frac{m_y(\bar{R}-1)}{\bar{R}-\bar{m}}]\mu_{x,t}$$

We define M and σ the same as before (equations 37 and 38). We also define D_x:

$$D_x = M[1 - \frac{m_y(\bar{R}-1)}{\bar{R}-\bar{m}}] \tag{44}$$

Notice that:

$$\begin{aligned}
&\frac{1}{1 - m_y\frac{\bar{R}-1}{\bar{R}}}(1 - \frac{\bar{m}}{\bar{R}})[1 - \frac{m_y(\bar{R}-1)}{\bar{R}-\bar{m}}] \\
&= \frac{1}{1 - m_y\frac{\bar{R}-1}{\bar{R}}}[1 - \frac{\bar{m}}{\bar{R}} - m_y\frac{\bar{R}-1}{\bar{R}}] \\
&= 1 - \frac{\frac{\bar{m}}{\bar{R}}}{1 - m_y\frac{\bar{R}-1}{\bar{R}}} \\
&= 1 - M
\end{aligned}$$

And that the following parameter values hold:

$$0 < M < 1$$

$$\sigma > 0$$

$$0 < D_x < M$$

Then:

$$x_t = -\sigma r_t + M\mathbb{E}_t[x_{t+1}] + (1 - M)\mu_{x,t} + D_x(\mu_{x,t} - \mathbb{E}_t[\mu_{x,t+1}]) \tag{45}$$

C.2 Determinacy of Long-Term Behavioral New Keynesian Model

$$x_t = -\sigma r_t + M\mathbb{E}_t[x_{t+1}] + (1 - M)\mu_{x,t} + D_x(\mu_{x,t} - \mathbb{E}_t[\mu_{x,t+1}])$$

$$\mu_{x,t+1} = \chi_y x_t + (1 - \chi_y)\mu_{x,t}$$

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t[\pi_{t+1}]$$

$$i_t = r_t + \mathbb{E}_t[\pi_{t+1}]$$

$$i_t = \phi_\pi \pi_t$$

We input the μ equations into the IS equation:

$$x_t = -\sigma r_t + M\mathbb{E}_t[x_{t+1}] + (1-M)\mu_{x,t} + D_x(\mu_{x,t} - (1-\chi_y)\mu_{x,t} - \chi_y x_t)$$

$$x_t = -\sigma r_t + M\mathbb{E}_t[x_{t+1}] + (1-M)\mu_{x,t} + D_x(\chi_y \mu_{x,t} - \chi_y x_t)$$

$$x_t = -\sigma r_t + (M - D_x\chi_y)\mathbb{E}_t[x_{t+1}] + (1 - M + D_x\chi_y)\mu_{x,t}$$

$$x_t = -\sigma r_t + \tilde{M}\mathbb{E}_t[x_{t+1}] + (1-\tilde{M})\mu_{x,t} \tag{46}$$

where:

$$\tilde{M} = M - D_x\chi_y$$

Inputting equation 44 yields:

$$\tilde{M} = M - M[1 - \frac{m_y(\bar{R}-1)}{\bar{R}-\bar{m}}]$$

$$= M\frac{m_y(\bar{R}-1)}{\bar{R}-\bar{m}}$$

Thus:

$$0 < \tilde{M} < M < 1$$

We can simplify the Fisher equation and Taylor rule (equation 12 and **??**) to get:

$$r_t = \phi_\pi \pi_t - \mathbb{E}_t[\pi_{t+1}] \tag{47}$$

We can express the NKPC (equation 21 as follows:

$$\mathbb{E}_t[\pi_{t+1}] = \frac{1}{\beta}\pi_t - \frac{\kappa}{\beta}x_t$$

Inputting equation 47 into equation 46 and rearranging yields:

$$x_t = -\sigma(\phi_\pi \pi_t - \mathbb{E}_t[\pi_{t+1}]) + \tilde{M}\mathbb{E}_t[x_{t+1}] + (1-\tilde{M})\mu_{x,t}$$

$$x_t = -\sigma(\phi_\pi \pi_t - \frac{1}{\beta}\pi_t + \frac{\kappa}{\beta}x_t) + \tilde{M}\mathbb{E}_t[x_{t+1}] + (1-\tilde{M})\mu_{x,t}$$

$$(1 + \sigma\frac{\kappa}{\beta})x_t = -\sigma(\phi_\pi - \frac{1}{\beta})\pi_t + \tilde{M}\mathbb{E}_t[x_{t+1}] + (1-\tilde{M})\mu_{x,t}$$

$$\mathbb{E}_t[x_{t+1}] = \frac{1}{\tilde{M}}(1+\sigma\frac{\kappa}{\beta})x_t + \frac{1}{\tilde{M}}\sigma(\phi_\pi - \frac{1}{\beta})\pi_t - \frac{1}{\tilde{M}}(1-\tilde{M})\mu_{x,t}$$

Then we can express the model in the following form:

$$\mathbb{E}_t[Z_{t+1}] = AZ_t$$

where:

$$Z_t = \begin{pmatrix} x_t \\ \pi_t \\ \mu_{x,t} \end{pmatrix}$$

$$A = \begin{pmatrix} \frac{1}{\tilde{M}}(1+\sigma\frac{\kappa}{\beta}) & \frac{1}{\tilde{M}}\sigma(\phi_\pi - \frac{1}{\beta}) & -\frac{1}{\tilde{M}}(1-\tilde{M}) \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ \chi_y & 0 & 1-\chi_y \end{pmatrix}$$

Next, we find the eigenvalues of A. We have one state in this system ($\mu_{x,t}$. For determinacy, we need that if controls deviate from their optimal level then expectations explode while if states deviate then expectations do not explode. So we need exactly one eigenvalue to be less than 1 in absolute value. Let's find the eigenvalues of A:

$$|A-\lambda I| = \begin{vmatrix} \frac{1}{\tilde{M}}(1+\sigma\frac{\kappa}{\beta}) - \lambda & \frac{1}{\tilde{M}}\sigma(\phi_\pi - \frac{1}{\beta}) & -\frac{1}{\tilde{M}}(1-\tilde{M}) \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} - \lambda & 0 \\ \chi_y & 0 & 1-\chi_y-\lambda \end{vmatrix}$$

$$= (-1)^2(\frac{1}{\tilde{M}}(1+\sigma\frac{\kappa}{\beta})-\lambda)(\frac{1}{\beta}-\lambda)(1-\chi_y-\lambda)+(-1)^3\frac{1}{\tilde{M}}\sigma(\phi_\pi-\frac{1}{\beta})(-\frac{\kappa}{\beta})(1-\chi_y-\lambda)+(-1)^4(-\frac{1}{\tilde{M}}(1-\tilde{M}))(-1)(\frac{1}{\beta}-\lambda)\chi_y$$

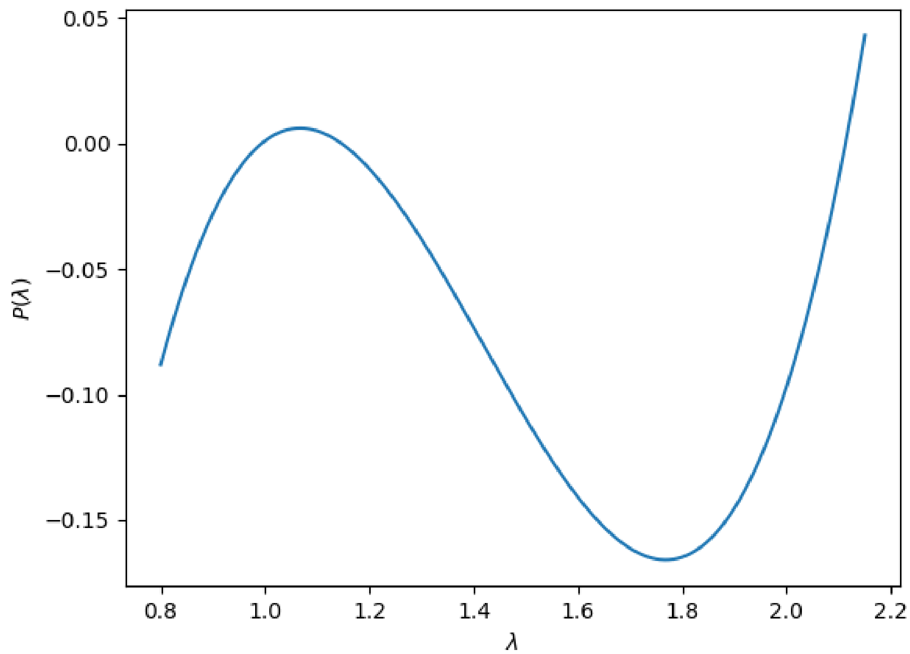
$$= (\frac{1}{\tilde{M}}(1+\sigma\frac{\kappa}{\beta}) - \lambda)(\frac{1}{\beta} - \lambda)(1-\chi_y-\lambda) + \frac{1}{\tilde{M}}\sigma(\phi_\pi - \frac{1}{\beta})(\frac{\kappa}{\beta})(1-\chi_y-\lambda) + (\frac{1}{\tilde{M}}(1-\tilde{M}))(\frac{1}{\beta}-\lambda)\chi_y$$

$$\begin{aligned} &= -\lambda^3+(\frac{1}{\tilde{M}}(1+\sigma\frac{\kappa}{\beta})+\frac{1}{\beta}+1-\chi_y)\lambda^2-[(\frac{1}{\tilde{M}}(1+\sigma\frac{\kappa}{\beta})(\frac{1}{\beta}+1-\chi_y)+\frac{1}{\beta}(1-\chi_y))]\lambda+\frac{1}{\tilde{M}}(1+\sigma\frac{\kappa}{\beta})\frac{1}{\beta}(1-\chi_y) \\ &\quad + \frac{1}{\tilde{M}}\sigma(\phi_\pi - \frac{1}{\beta})(\frac{\kappa}{\beta})(1-\chi_y) - \frac{1}{\tilde{M}}\sigma(\phi_\pi - \frac{1}{\beta})(\frac{\kappa}{\beta})\lambda + (\frac{1}{\tilde{M}}(1-\tilde{M}))(\frac{1}{\beta})\chi_y - (\frac{1}{\tilde{M}}(1-\tilde{M}))\chi_y\lambda \end{aligned}$$

Multiplying by -1 and defining a characteristic equation:

$$\begin{aligned} P(\lambda) &= \lambda^3 - (\frac{1}{\tilde{M}}(1+\sigma\frac{\kappa}{\beta}) + \frac{1}{\beta} + 1 - \chi_y)\lambda^2 \\ &\quad + [\frac{1}{\tilde{M}}(1+\sigma\frac{\kappa}{\beta})(\frac{1}{\beta}+1-\chi_y) + \frac{1}{\beta}(1-\chi_y) + \frac{1}{\tilde{M}}\sigma(\phi_\pi - \frac{1}{\beta})(\frac{\kappa}{\beta}) + (\frac{1}{\tilde{M}}(1-\tilde{M}))\chi_y]\lambda \\ &\quad - [\frac{1}{\tilde{M}}(1+\sigma\frac{\kappa}{\beta})\frac{1}{\beta}(1-\chi_y) + \frac{1}{\tilde{M}}\sigma(\phi_\pi - \frac{1}{\beta})(\frac{\kappa}{\beta})(1-\chi_y) + (\frac{1}{\tilde{M}}(1-\tilde{M}))(\frac{1}{\beta})\chi_y] \end{aligned}$$

Figure 4: Characteristic Function



We can plot the characteristic function P for different values of λ in figure 4 (we are not making a graphical proof - this is just for reference).

From the polynomial, we see that when $\lambda \leq 0$, the function must be strictly negative so there are no eigenvalues that are nonpositive.. Therefore, if $P(1) < 0$ then we know that there must be an even number of points such that $\lambda < 1$ and $P(\lambda) = 0$. If $P(1) > 0$ then we know that there must be an odd number of points such that $\lambda > 1$ and $P(\lambda) = 0$. Therefore, a necessary but not sufficient condition for there to be exactly one eigenvalue that is less than 1 in absolute value is:

$$P(1) > 0$$

We analyse the condition $P(1) > 0$:

$$\begin{aligned} P(1) = 1 - (&\frac{1}{\tilde{M}}(1+\sigma\frac{\kappa}{\beta}) + \frac{1}{\beta} + 1 - \chi_y) \\ &+ [\frac{1}{\tilde{M}}(1+\sigma\frac{\kappa}{\beta})(\frac{1}{\beta} + 1 - \chi_y) + \frac{1}{\beta}(1-\chi_y) + \frac{1}{\tilde{M}}\sigma(\phi_\pi - \frac{1}{\beta})(\frac{\kappa}{\beta}) + (\frac{1}{\tilde{M}}(1-\tilde{M}))\chi_y] \\ &- [\frac{1}{\tilde{M}}(1+\sigma\frac{\kappa}{\beta})\frac{1}{\beta}(1-\chi_y) + \frac{1}{\tilde{M}}\sigma(\phi_\pi - \frac{1}{\beta})(\frac{\kappa}{\beta})(1-\chi_y) + (\frac{1}{\tilde{M}}(1-\tilde{M}))(\frac{1}{\beta})\chi_y] \end{aligned}$$

$$= 1 - (\frac{1}{\beta} + 1 - \chi_y)$$
$$+ [\frac{1}{\tilde{M}}(1 + \sigma\frac{\kappa}{\beta})(-\chi_y + \frac{1}{\beta}\chi_y) + \frac{1}{\beta}(1 - \chi_y) + \frac{1}{\tilde{M}}\sigma(\phi_\pi - \frac{1}{\beta})(\frac{\kappa}{\beta})\chi_y + (\frac{1}{\tilde{M}}(1 - \tilde{M}))\chi_y(1 - \frac{1}{\beta})]$$

$$= \chi_y$$
$$+ [\frac{1}{\tilde{M}}(1 + \sigma\frac{\kappa}{\beta})(-\chi_y + \frac{1}{\beta}\chi_y) + \frac{1}{\beta}(-\chi_y) + \frac{1}{\tilde{M}}\sigma(\phi_\pi - \frac{1}{\beta})(\frac{\kappa}{\beta})\chi_y + (\frac{1}{\tilde{M}}(1 - \tilde{M}))\chi_y(1 - \frac{1}{\beta})]$$

$$\begin{aligned}
&= \chi_y[1 - \frac{1}{\beta} + \frac{1}{\tilde{M}}(1 + \sigma\frac{\kappa}{\beta})(-1 + \frac{1}{\beta}) + \frac{1}{\tilde{M}}\sigma(\phi_\pi - \frac{1}{\beta})(\frac{\kappa}{\beta}) + (\frac{1}{\tilde{M}}(1 - \tilde{M}))(1 - \frac{1}{\beta})] \\
&= \chi_y[1 - \frac{1}{\beta} + \frac{1}{\tilde{M}}[-1 + \frac{1}{\beta} - \sigma\frac{\kappa}{\beta} + \sigma\frac{\kappa}{\beta^2} + \sigma\phi_\pi\frac{\kappa}{\beta} - \sigma\frac{\kappa}{\beta^2}] + (\frac{1}{\tilde{M}}(1 - \tilde{M}))(1 - \frac{1}{\beta})] \\
&= \chi_y[1 - \frac{1}{\beta} + \frac{1}{\tilde{M}}[-1 + \frac{1}{\beta} - \sigma\frac{\kappa}{\beta} + \sigma\phi_\pi\frac{\kappa}{\beta}] + (\frac{1}{\tilde{M}}(1 - \tilde{M}))(1 - \frac{1}{\beta})] \\
&= \frac{\chi_y}{\beta}[[1 - \frac{1}{\tilde{M}} + \frac{1}{\tilde{M}}(1 - \tilde{M})](\beta - 1) + \frac{1}{\tilde{M}}[-\sigma\kappa + \sigma\phi_\pi\kappa]] \\
&= \frac{\chi_y}{\tilde{M}\beta}\kappa\sigma(\phi_\pi - 1)
\end{aligned}$$

We see that $P(1)$ is only greater than 0 when $\phi_\pi > 1$. So we observe that we need to get one and only one eigenvalue that is less than 1 is absolute value for determinacy, this requires that $P(1) > 0$ and this only occurs when $\phi_\pi > 1$. Therefore, we require $\phi_\pi > 1$ for determinacy.

C.3 Costs of Zero Lower Bound in Long-Term Behavioral New Keynesian Model

We provide a sketch proof.

If $\bar{\mu}_{\hat{x}} = 0$ then:

$$\bar{\mu}_{\hat{x}} = M\bar{\hat{x}} - \sigma(0.01 - \frac{\kappa}{1-\beta}\bar{\hat{x}}) \tag{48}$$

Rearranging:

$$\left[1 - M - \frac{\kappa}{1-\beta}\right]\bar{\hat{x}} = -\sigma(0.01)$$

However, this implies that the long-term expectatons of $\hat{x}$ should actually be:

$$\bar{\mu}_{\hat{x}} = -\frac{1}{1 - M - \frac{\kappa}{1-\beta}}\sigma(0.01)$$

And if we input this into equation 48 then we'll get that $\bar{\hat{x}}$ is lower and thus $\bar{\mu}_{\hat{x}}$ falls. And this continues indefinitely.

We can also show this by noting that there is no possible steady state available.

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