

# The Inflation Target and the Equilibrium Real Rate

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## Abstract

Many economists have proposed raising the inflation target to reduce the probability of hitting the zero lower bound (ZLB). It is both widely assumed and a feature of standard models that raising the inflation target does not impact the equilibrium real rate. I demonstrate that once heterogeneity is introduced, raising the inflation target causes the equilibrium real rate to fall in the New Keynesian model. This implies that raising the inflation target will increase the nominal interest rate by less than expected and thus will be less effective in reducing the probability of hitting the ZLB. The channel is that a rise in the inflation target lowers the average markup by price rigidities and a fall in the average markup lowers the equilibrium real rate by household heterogeneity which could come from overlapping generations or idiosyncratic labor shocks. Raising the inflation target from 2% to 4% lowers the equilibrium real rate by 0.38 percentage points in my baseline calibration. I also analyse the optimal inflation level and provide empirical evidence in support of the model mechanism.

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<sup>†</sup>Link: [www.cdcotton.com/papers/pistar-rstar.pdf](http://www.cdcotton.com/papers/pistar-rstar.pdf)

# 1 Introduction

Many economists have proposed raising the inflation target to reduce the probability of hitting the zero lower bound (ZLB). Nearly all developed countries were constrained by the ZLB during the financial crisis. Moreover, it is widely believed that average real interest rates have fallen.<sup>1</sup> This implies that average nominal interest rates will be lower going forward. Consequently, there has been a re-evaluation of the risk that central banks will hit the ZLB. Hitting the bound is bad for economic outcomes because central banks have less room to lower nominal interest rates and stimulate the economy during bad times. Therefore many economists (including [Blanchard et al. \(2010\)](#), [Ball \(2014\)](#), [Krugman \(2014\)](#)) have proposed raising the inflation target from the standard objective of 2% to 4% claiming this will raise average nominal interest rates and thus reduce the probability of hitting the ZLB.

It is widely assumed that raising the inflation target will not affect the equilibrium real rate. The equilibrium real (nominal) rate is the real (nominal) interest rate on short-term safe assets when there are no shocks. Standard macroeconomic models very commonly assume flexible prices and/or a representative agent. With either of these assumptions, the equilibrium real rate is unaffected by changing average inflation. This is also a historic concept introduced by [Fisher \(1907\)](#) and is often taken for granted within policy discussions. For example, [Ball \(2014\)](#) states that the long run level of the real interest rate is “independent of monetary policy”. Thus, it is widely believed that raising the inflation target by 2p.p. will have no impact upon the equilibrium real rate and will therefore raise the equilibrium nominal rate by a corresponding 2p.p.

My primary contribution is to demonstrate a new channel by which raising the inflation target will lower the equilibrium real rate. Once I account for household heterogeneity (through either overlapping generations or idiosyncratic risk) within the standard New Keynesian model, I find that raising the inflation target lowers the equilibrium real rate. This implies that a rise in the inflation target will raise the average nominal interest rate by less than expected. Since nominal interest rates will rise by less than expected, raising the inflation target will reduce the probability of avoiding the ZLB by less than is commonly believed. The channel has two stages. Firstly, price rigidities imply that a rise in the inflation target lowers the markup. Secondly, household heterogeneity implies that a fall in the markup lowers the equilibrium real rate.

The first part of the channel is a standard, albeit often overlooked, feature of New Keynesian models. A firm’s markup is just the ratio of its price to its nominal marginal cost. When firms set their prices infrequently, a higher average inflation level has two opposing impacts upon average markups. Firstly, higher inflation means that when a firm does not reset its price then its markup falls by relatively more since with higher inflation nominal marginal costs rise relatively more quickly. Secondly, firms observe that their markups fall more quickly and therefore set

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<sup>1</sup>[Holston et al. \(2017\)](#) estimate that it has fallen by an average of 2.3p.p. since 1990 (across the US, Canada, the Euro Area and the UK). Recent estimates of the US equilibrium real rate by [Negro et al. \(2017\)](#), [Holston et al. \(2017\)](#), [Johannsen and Mertens \(2016\)](#), [Kiley \(2015\)](#), [Laubach and Williams \(2015\)](#), [Lubik and Matthes \(2015\)](#) lie between 0.1% and 1.8%.

their markup to be higher when they do get to reset their prices. It can be shown that with no discounting these two effects cancel out and thus average markups are unchanged by raising average inflation. However, with discounting, the first effect dominates since firms care more about making profits in the current period and so do not want to set their current markup to be very high when they reset their price. Therefore, a rise in average inflation lowers the average markup.

The second part of the channel is that once you allow for household heterogeneity a fall in the markup lowers the equilibrium real rate. Taking the example of heterogeneity through overlapping generations (OLG): A fall in the markup lowers firm profits and thus reduces the value of shares and of overall savings. A fall in the amount of savings *ceteris paribus* lowers the consumption of the old relative to the young. This means the old have higher marginal utility from consuming than the young. Thus, there is greater competition among young people to save for when they are old and so the price of savings rises. As the price of savings rises, the return on savings (the equilibrium real rate) falls. To my knowledge, this part of the channel has not been covered in the literature.

This contrasts with a representative agent New Keynesian model where a fall in the markup has no impact on the equilibrium real rate. In this case, a rise in inflation still lowers the markup which in turn lowers firm profits and thus reduces the value of shares. However, within a representative agent framework, the agent's consumption path does not depend upon average household savings since it does not matter what level of savings the agent chooses to hold during their infinitely long life. Instead, without shocks, they just set their level of consumption to be the same over time and thus the equilibrium real rate is purely determined by the agent's patience.

I estimate the impact of the channel through a fully calibrated model. I study the effect of raising the inflation target within a model with standard New Keynesian features and a fully calibrated life cycle framework. Within the baseline calibration, I find the equilibrium real rate falls by 0.38*p.p.* when the inflation target is raised from 2% to 4%. This implies that average nominal interest rates rise by 1.62*p.p.* as opposed to the 2*p.p.* that would typically be expected and be found within standard models. Thus, raising the inflation target mitigates the probability of hitting the ZLB by less than expected. When I reduce the intertemporal elasticity of substitution from 0.5 to 0.1<sup>2</sup>, I find the fall in the equilibrium real rate increases to 0.67*p.p.*

I compute the optimal change in inflation in response to a fall in the equilibrium real rate as well as the optimal level of inflation more generally. To assess the optimal inflation target, I find the welfare of the simulated path of the economy of my model under different inflation targets taking into account the ZLB with calibrated shocks. Much of the motivation for raising the inflation target is based upon the suggestion that the equilibrium real rate has fallen. I assess how much the inflation target increases when the equilibrium real rate falls by 2*p.p.* In my baseline calibration, I find this increases the optimal inflation target by 0.3*p.p.* When I allow for larger shocks which increase the probability of hitting the ZLB, I find the increase in the optimal inflation

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<sup>2</sup>Recent research by [Best et al. \(2018\)](#) suggests an intertemporal elasticity of substitution of 0.1.

target is 0.6p.p. I also analyze the level of the optimal inflation target. The optimal inflation target is always around 1p.p. This is similar to [Coibion et al. \(2012\)](#) who assess the optimal inflation target in a representative agent model. Therefore, the benefits of avoiding the ZLB appear to be dominated by the welfare costs of price dispersion even for relatively low inflation targets.

I also provide empirical evidence for my mechanism. I show there is a negative empirical relationship between long-run inflation and the equilibrium real rate which supports my hypothesized channel. In recent years, inflation and the real interest rate have both fallen across developed countries. This would contradict my channel if the fall in inflation was the only change that could have driven real interest rates lower.<sup>3</sup> However, many factors have been proposed that have lowered real interest rates for other reasons across developed countries.<sup>4</sup> Indeed, it is puzzling that real interest rates have not fallen by more. [Gagnon \(2009\)](#) argue that demographic factors alone can explain the fall in the equilibrium real rate while [Eggertsson et al. \(2017\)](#) argue that real rates should be much lower. I take this into account in my empirical analysis by looking at panel data regressions of the real rate on long-run inflation controlling for country and time fixed effects in OECD countries. The time fixed effects allow me to control for any common change in real rates across countries. I find a 1p.p. rise in long-run inflation lowers the equilibrium real rate by 0.61p.p.

There is also a negative empirical relationship between long-run inflation and the markup. I am interested in this relationship because the first part of my channel is that a rise in inflation lowers the markup. Using the labor share as a proxy for the inverse of the markup, I conduct similar panel data regressions to the real rate on long-run inflation case. I find a 1p.p. rise in long-run inflation lowers the long-run markup by at least 0.46p.p.

There is a historical literature that looks at the impact of inflation on the equilibrium real rate through non-interest paying money balances but it may be less relevant today. [Mundell \(1963\)](#) and [Tobin \(1965\)](#) argued that when inflation rises, it becomes costlier to hold money so agents save more in capital, leading to a fall in the equilibrium real rate. A key assumption of this literature is that money does not pay interest. This has two important implications. Since money does not pay interest, agents need some other incentive to hold money such as the assumption of a cash-in-advance constraint or money-in-utility. Secondly, most central banks in developed countries have now shifted to a framework where they control nominal interest rates by paying interest on reserves in which case a rise in inflation will lead to higher interest on reserves and thus no portfolio shift to capital away from money. Therefore, this literature appears less relevant to modern central banking. My proposed channel is very different because it does not rely upon money holdings in any way.<sup>5</sup>

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<sup>3</sup>My channel would predict a rise in the equilibrium real rate when inflation falls.

<sup>4</sup>Stories include: demographic changes ([Carvalho et al. \(2016\)](#), [Gagnon et al. \(2016\)](#)), global savings glut ([Caballero and Farhi \(2017\)](#)), secular stagnation ([Eggertsson and Mehrotra \(2014\)](#)), low productivity growth ([Yi and Zhang \(2017\)](#)), high inequality ([Lancastre \(2018\)](#)).

<sup>5</sup>There are many other papers in this literature. For instance, [Stockman \(1981\)](#) proposed a reverse-Mundell-Tobin effect in which raising inflation raises the equilibrium real rate due to a cash in advance constraint on investment i.e. you need to take money out of the bank a period before investing.

My model relates to several interesting literatures on heterogeneous agent models: 1. the allocation of profits, 2. redistributive effects of monetary policy, 3. optimal monetary policy with heterogeneous agents. Unlike many heterogeneous agent models, I allow for the endogenous allocation of profits. Most heterogeneous agent models exogenously allocate profits i.e. certain agents are assigned to receive profits. For instance, [Werning \(2015\)](#) considers how these exogenous profit allocations impact the marginal propensity to consume and related implications. I instead consider the case where agents only receive profits by owning shares in firms which get traded each period. Thus, it is an endogenous feature of my model that old people naturally consume less as a result of the fall in the markup.

Raising the inflation target within a heterogeneous agent model can generate interesting long-run distributional effects. Raising the inflation target can have short-term redistributive effects which hurt savers and benefit borrowers by lowering the value of nominal assets. [Doepke et al. \(2015\)](#) consider these short-term redistributive effects in detail. My paper implies that there can actually be long-run distributional effects as well. A rise in the inflation target reduces profits and thus the value of shares and total savings. This implies that old people, who rely upon savings, consume relatively less and young people consume relatively more indefinitely as a result of a rise in the inflation target.

I contribute to the literature on optimal monetary policy in heterogeneous agent models. I investigate optimal monetary policy within a New Keynesian model with OLG features. [Lepetit \(2017\)](#) shows that within a New Keynesian model with perpetual youth, it can be optimal to set a positive inflation target because heterogeneity can imply that private discounting is higher than social discounting. In this case, central banks raise inflation to lower average markups. My paper is quite different because the primary reason central banks want to raise inflation above zero is to avoid hitting the ZLB which Lepetit does not consider.

There is empirical evidence that supports my channel. Other papers have suggested that raising the long-run inflation rate lowers the long-run real interest rate. [King and Watson \(1997\)](#) consider the impact of raising inflation upon the real interest rate and show that an increase in long-run inflation leads to a decrease in the long-run real interest rate regardless of the restrictions imposed in a structural VAR model for US data. They find that a rise in of 1p.p. in long-run inflation lowers the equilibrium real rate by 0.66p.p. [Rapach \(2003\)](#) extends the analysis to 14 countries with a richer structural model. He demonstrates that a rise in long-run inflation leads a fall of between 0.94p.p. and 0.59p.p. in the equilibrium real rate.

Other papers have also suggested that raising the long-run inflation rate lowers the markup, which supports the first stage of the mechanism in my model. [Bénabou \(1992\)](#) finds that raising inflation by 1p.p. lowers the markup by 0.36p.p. using a relatively reduced form approach with just US data. [Banerjee and Russell \(2001\)](#) apply a structural VAR approach to the G7 countries and Australia. They find that a 1p.p. rise in annual steady state inflation generates a fall of between 0.3p.p. and 2p.p. in the long-run markup.

In section 2, I outline a simple model that captures the key features found in the rest of the

paper. I then outline the full model (section 3). I discuss the model solution and calibration in section 4. I use the full model to analyse how changing the inflation target will impact the equilibrium real rate in section 5. I then consider the optimal inflation target in section 6. I discuss my supporting empirical results in section 7. Section 8 concludes.

## 2 Intuition through a Simplified Model

I break the intuition for the channel into two parts. First, it is demonstrated that a rise in inflation lowers the average markup through firms' pricing decisions. Next, it is shown that a fall in the markup lowers the equilibrium real rate through multiple forms of household heterogeneity.

### 2.1 Relationship between the Inflation Level and the Markup

A firm's markup, denoted  $m_t$ , is its current price,  $P_t^*$ , divided by its nominal marginal cost,  $MC_t$ :

$$m_t = \frac{P_t^*}{MC_t}$$

Firms' profits depend upon their markup. If they set their markup too high, they will not make enough sales. If they set it too low, they will make a lot of sales but with too little profit on each sale. When firms have fully flexible prices, they can set their price so that their markup yields the maximum profits each period. In the common case where firms face constant elasticity of demand, the optimal markup is just  $\frac{\sigma}{\sigma-1}$  where  $\sigma$  is the CES parameter.

Setting markups is more complex in the case with infrequent price adjustment. When firms can only change their price infrequently, they are no longer able to set the optimal flexible price markup each period. In the case of positive inflation: There will be two important effects. Firstly, if firms do not get to change their price in a period then their markup will fall. This is because their nominal marginal costs ( $MC_t$ ) rise (due to the rise in the price level) while their price ( $P_t^*$ ) remains constant. Secondly, in anticipation that they may not get to change their price in the future and thus their markup will fall, firms will set their markups to be higher than the optimal flexible price markup when they do get to change their price.

The impact of raising inflation on the markup depends upon the degree of discounting. In the case with no discounting, firms will weight their profits equally in current and future periods. This leads to a special case where the markup is unaffected by changing the level of inflation since the two effects on the markup cancel out. However, when firms discount the future, they will weight their current period markup more in their decision-making. This implies that they set a lower markup when they get to change their price and thus that the average markup is lower with positive inflation. As the level of inflation rises, the strength of this effect will increase.

For example, in the case of Calvo pricing: Denote  $\sigma$  to be the elasticity of substitution between goods,  $\beta$  to be the discount factor,  $\lambda$  to be the probability with which firms can update their price each period and  $\bar{\Pi}$  to be the steady state level of gross inflation (i.e.  $\frac{P_{t+1}}{P_t}$ ). Then the firms steady

state average markup  $\bar{m}$  is given by:<sup>6</sup>

$$\bar{m} = \frac{\sigma}{\sigma - 1} \left[ \frac{\bar{\Pi}^{1-\sigma} - (1 - \lambda)\beta}{\bar{\Pi}^{1-\sigma} - (1 - \lambda)} \right] \left[ \frac{\bar{\Pi}^{-\sigma} - (1 - \lambda)}{\bar{\Pi}^{-\sigma} - (1 - \lambda)\beta} \right] \quad (1)$$

When  $\beta = 1$  so there is no discounting, equation 1 simplifies to give  $\bar{m} = \frac{\sigma}{\sigma-1}$  so raising inflation has no impact upon the markup and firms will set their markup to be at the same level as without price stickiness. However, when  $\beta < 1$  so there is discounting, raising inflation always lowers the markup. This is easy to see in the extreme case of full discounting when  $\beta = 0$  in which case equation 1 simplifies to:<sup>7</sup>

$$\bar{m} = \frac{\sigma}{\sigma - 1} \frac{\bar{\Pi}^{1-\sigma} - (1 - \lambda)\bar{\Pi}}{\bar{\Pi}^{1-\sigma} - (1 - \lambda)}$$

The frequency of price changes does not affect the relationship at low levels of inflation. If the frequency with which firms adjust prices increases, this would reduce the feedback from inflation to the markup.<sup>8</sup> However, [Gagnon \(2009\)](#) demonstrates that the frequency with which firms change their price does not appear to vary below annual rates of inflation of 10%. This makes sense because firms are likely to change their price for other reasons (like idiosyncratic demand or costs) than just inflation so the frequency of price changes does not need to change with low inflation.

The negative inflation-markup relationship also holds with price rigidities based upon adjustment costs. The relationship would hold in the case of menu costs (fixed costs of updating prices) or Rotemberg costs (convex adjustment costs of updating prices). The intuition is that firms prefer to pay the cost of updating their price in the future (with positive discounting) so they set a lower markup when inflation rises.

This is a general result. To get a negative relationship between inflation and the markup, I require that firms set their prices infrequently and discount the future. [Nakamura and Steinsson \(2008\)](#) demonstrates that firms have low frequencies of price changes. [Jagannathan et al. \(2016\)](#) demonstrates that firms discount the future significantly. It is also worth stressing that this relationship is present in the representative agent New Keynesian model - nothing here depends upon household heterogeneity.

## 2.2 Relationship between the Markup and the Equilibrium Real Rate

**Simple Model** I have shown that markup is determined by the level of inflation so I take the markup as given and concentrate upon the real side of a simple model.

<sup>6</sup>The derivations are shown in appendix A.1.

<sup>7</sup>I prove that when steady state inflation rises the markup falls for all  $\beta < 1$  in appendix A.1.

<sup>8</sup>This occurs because firms would set their markup for shorter periods of time on average which means that the markup would fall by less before being changed for a given level of inflation.



Firm's produce using a linear production function.<sup>9</sup> Therefore, output  $Y_t$  equals labor  $L_t$ :

$$Y_t = L_t \quad (2)$$

The real marginal cost of firms  $MC_t$  will just be the real wage  $W_t$ :

$$MC_t = W_t \quad (3)$$

The markup  $m_t$  is just the price divided by the nominal marginal cost which, by definition, equals the inverse of the real marginal cost ( $\frac{1}{MC_t}$ ) so we can rewrite the marginal cost wage (equation 3) relationship as:

$$\frac{1}{m_t} = W_t \quad (4)$$

The total real profits  $\Omega_t$  of firms will just be their real sales which is just their output minus their costs of labor:

$$\Omega_t = Y_t - W_t L_t$$

$Y_t$  can be substituted out with  $L_t$  (equation 2) and then multiply and divide the first term on the RHS by  $W_t$  to give:

$$\Omega_t = \left(\frac{1}{W_t} - 1\right) W_t L_t$$

I then apply the markup-wage relationship (equation 4):

$$\Omega_t = (m_t - 1) \frac{W_t}{P_t} L_t \quad (5)$$

**Asset Supply** I break down the solution into the supply and demand for assets. Asset supply is the amount of assets that are available for households to hold. Asset demand is the amount of assets that agents want to hold.

The only asset that agents can save in is shares in firms. The total real value of firms is denoted by  $Z_t$ . Therefore, asset supply, denoted  $A^s$ , is given by:

$$A^s = Z \quad (6)$$

By standard asset pricing, we know that the price of buying  $b$  shares  $bZ_t$  must equal the next period return on those shares discounted at  $r_{t+1}$ . The next period return of those shares is the dividends received from profits ( $b\Omega_{t+1}$ ) plus the price the shares are sold for at  $t + 1$  ( $bZ_{t+1}$ ). Therefore:

$$Z_t = \frac{\Omega_{t+1} + Z_{t+1}}{1 + r_{t+1}} \quad (7)$$

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<sup>9</sup>Here we have effectively assumed that firms do not face price dispersion. This would be true, for example, under Rotemberg Pricing. The case with price dispersion generates exactly the same equations but is a little bit more complicated to derive. The results are shown in appendix A.2.



In steady state, we can rewrite equation 7 as:

$$\bar{Z} = \frac{\bar{\Omega}}{\bar{r}} \quad (8)$$

We can substitute the value of shares with profits (equation 8) in the asset supply equation (equation 6):

$$\bar{A}^s = \frac{\bar{\Omega}}{\bar{r}} \quad (9)$$

We can then substitute out profits using equation 5:

$$\bar{A}^s = \frac{\bar{m} - 1}{\bar{r}} \bar{W} \bar{L} \quad (10)$$

To make the problem simpler, we define relative assets  $a$  which are assets in terms of labor income:

$$a = \frac{A}{WL} \quad (11)$$

The supply of assets (equation 10) can be rewritten in relative terms to get:

$$\bar{a}^s = \frac{\bar{m} - 1}{\bar{r}} \quad (12)$$

Two features can be observed. Firstly, in equation 12 a rise in  $\bar{r}$  lowers  $\bar{a}^s$ . This makes sense because higher discounting implies the discounted sum of profits is lower so the value of firms falls. Equation 12 is plotted in figure 1. The blue curve represents  $\bar{a}^s$  with  $\bar{m} = 1.3$  and the orange curve represents  $\bar{a}^s$  with  $\bar{m} = 1.2$ . Since raising  $\bar{r}$  lowers  $\bar{a}^s$ , the curve has a downward slope. It may appear strange that the supply curve is downward sloping but this is because the vertical axis is the return on assets. The return on assets is like the inverse of the price of assets (since as the price of assets rises, the return agents make on those assets falls). If the curve was drawn with the price of assets on the vertical axis, it would have the usual upward sloping supply curve.

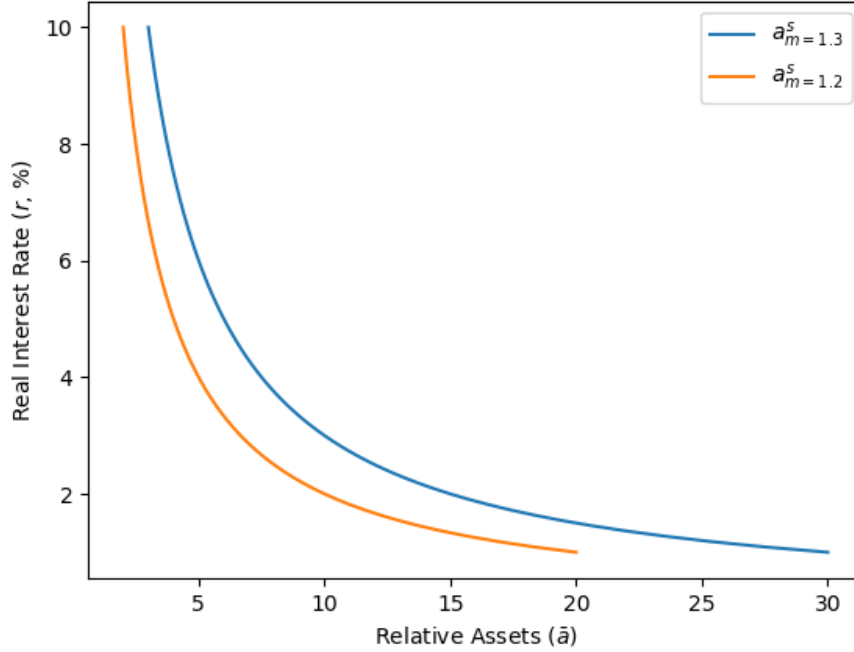
Secondly, observe that in equation 12 a fall in the markup  $\bar{m}$  lowers the relative asset supply  $\bar{a}^s$  for any real interest rate  $\bar{r}$ . This makes sense because when the markup falls, the value of firms falls and thus the value of owning shares in firms falls. This can also be seen in figure 1. Observe that the fall in the markup shifts the relative asset supply curve left from the blue curve with markup 1.3 to the orange curve with markup 1.2.

**Asset Demand: 1. Representative Agent** Next, I consider the shape of the asset demand under three different household structures: 1. Representative agent. 2. Heterogeneity through overlapping generations. 3. Heterogeneity through idiosyncratic labor.

In all standard representative agent problems, we derive an Euler condition of similar form to the following (I assume log utility to keep things very simple):

$$C_{t+1} = \beta(1 + r_{t+1})C_t \quad (13)$$

Figure 1: Relative Asset Supply under a Fall in the Markup



A steady state equilibrium requires that a representative agent consumes the same amount over time. If  $C_{t+1}$  is more (less) than  $C_t$  the Euler condition requires that  $1 + r_{t+1}$  is more (less) than  $\beta$ . Therefore, the only way we can have a steady state is when  $\beta(1 + r_{t+1})$  is stable which requires:

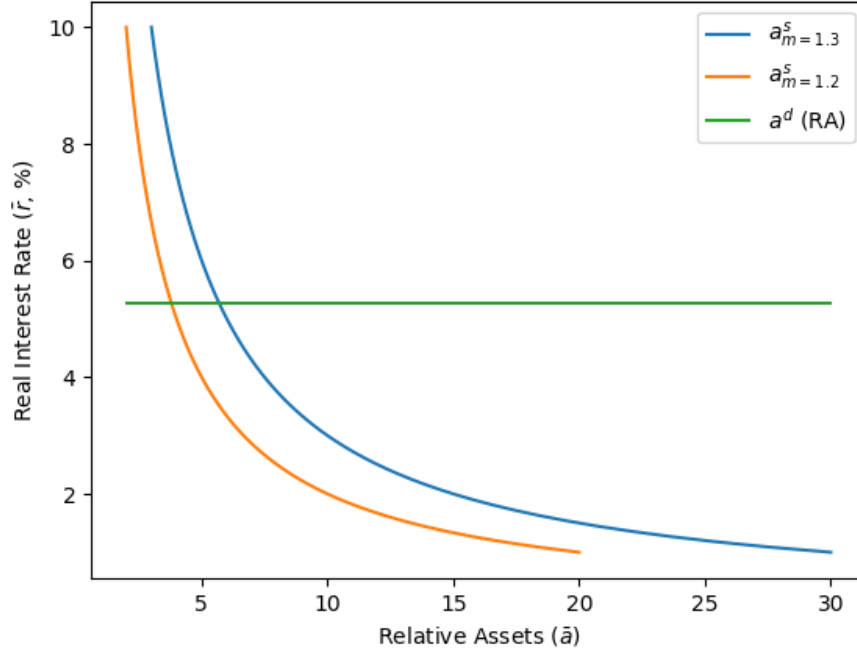
$$\bar{r} = \frac{1}{\beta} - 1 \quad (14)$$

Equation 14 is plotted in figure 2. Like in figure 1, the impact of a fall in the markup is considered. Observe that the asset demand is just a horizontal line since  $\bar{r}$  is always pinned down. Thus, a shift left in the supply of assets lowers the amount of assets held by the household but has no impact upon  $\bar{r}$ .

The reason the equilibrium real rate is unchanged is because in steady state the path of consumption of the agent must always be flat i.e.  $C_t = C_{t+1}$  by the Euler condition. Relative asset demand always adjusts to ensure this holds. Therefore, changing the assets held by the household cannot disturb the path of consumption of the agent. Thus, the marginal utility of the agent must always be flat i.e.  $u'(C_t) = u'(C_{t+1})$  regardless of changes in the supply of assets. The only way the marginal utility of the agent can be flat is if  $\bar{r}$  remains the same over time by the Euler condition.

**Asset Demand: 2. Overlapping Generations** Now, household heterogeneity is introduced. The implication in both cases of household heterogeneity that are considered is that the level of assets does impact the path of the household's marginal utility over time, meaning that the equilibrium

Figure 2: Equilibrium under a Fall in the Markup: 1. Representative Agent



real rate will be impacted by changing the markup.

I first consider a simple overlapping generations model based upon (Diamond, 1965). Every period a new generation is born. Each generation lives for two periods and then dies. The utility of a young agent is given by:

$$\log(C_{1,t}) + \beta \log(C_{2,t+1}) \quad (15)$$

Log utility is used for simplicity. Young agents work  $L$  unit and devote their income to either consumption  $C_{1,t}$  or asset purchases  $A_{t+1}$ :

$$C_{1,t} + A_{t+1} = W_t \quad (16)$$

Old agents merely consume  $C_{2,t+1}$  from their available assets. Their available assets are their assets from when they were young on which they have earned a return of  $r_{t+1}$ :

$$C_{2,t+1} = (1 + r_{t+1})A_{t+1} \quad (17)$$

The amount the young save can be solved for by inputting equations 16 and 17 into equation 15 and then taking first-order conditions. This yields:

$$A_{t+1} = \frac{\beta}{1 + \beta} W_t L \quad (18)$$

So agents save some constant fraction of their income each period. It is simpler to rewrite the agent's demand for assets in relative terms so equation 18 is divided by labor income and also written in steady state terms to yield:

$$\bar{a}^d = \frac{\beta}{1 + \beta} \quad (19)$$

In this case, the demand for savings is perfectly inelastic to changes in  $\bar{r}$ . This is something of a special case (due to log utility and only having two periods). In the full model, demand for relative assets is not perfectly inelastic. However, the generation structure in the full model is still such that the elasticity of demand is not perfectly elastic and thus the real interest rate changes in response to a shift left in the demand for assets.

Equation 19 is plotted in figure 3 where the impact of a fall in the markup is considered (as in figure 1). Observe that the asset demand is just a vertical line since  $\bar{a}^d$  is fixed. Thus, a shift left in the supply of assets lowers  $\bar{r}$  but has no impact upon the amount of assets demanded by the agent. The relative asset supply on this graph looks a bit different to previous asset demand/supply graphs since each period represents a generation and lasts for 25 – 30 years so it is necessary to rescale the curves to get back to an annual basis.<sup>10</sup>

This is effectively the opposite to the representative agent case. The reason the impact is so different is that a fall in the amount of savings held by the consumer affects the marginal utility of consumption of the young compared to the old. When assets fall, the old consume less relative to the young *ceteris paribus*. Thus, old people have a relatively higher marginal utility. Therefore, the price of assets rises since young agents are keener to save assets for when they are old. Consequently, the equilibrium real rate falls.

**Asset Demand: 3. Idiosyncratic Labor** Within this paper, household heterogeneity is primarily introduced through overlapping generations. However, an extension with idiosyncratic labor is considered and it is worthwhile to demonstrate that a similar intuition explains why the channel holds in this case.

There are many agents, each denoted with subscript  $i$ . Agents live forever and maximise their lifetime utility:

$$\max \sum_{t=0}^{\infty} \mathbb{E}_0[\beta^t u(C_{i,t})]$$

Agents receive a wage  $W$  from the amount they work  $L_{i,t}$ , which varies over time and across agents, and some real return  $r$  on assets  $A_{i,t}$ . Agents spend their money on consumption  $C_{i,t}$  and assets for the next period. Note that there are no aggregate shocks hence why  $W, r$  have no time subscripts. Their budget constraint is:

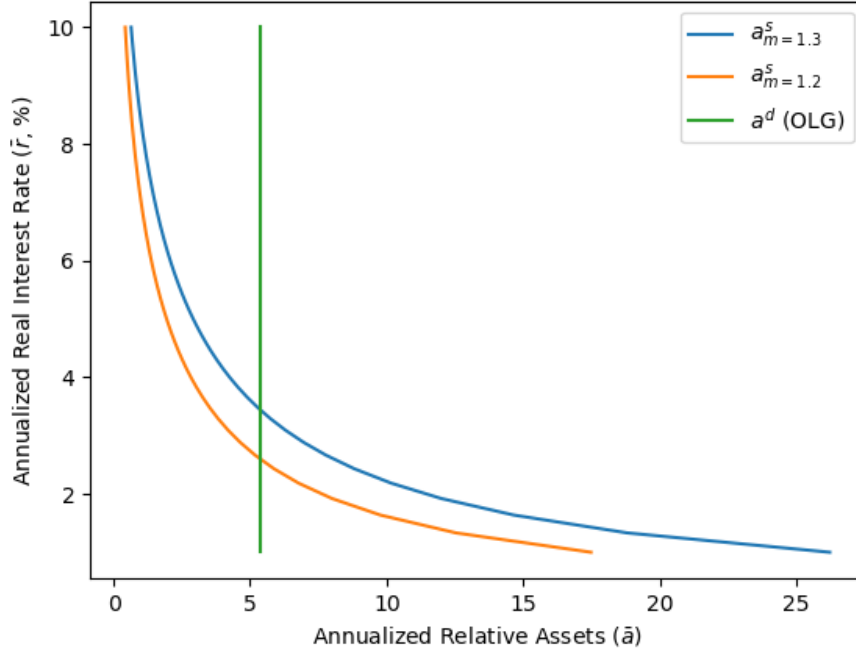
$$C_{i,t} + A_{i,t+1} = (1 + r)A_{i,t} + WL_{i,t}$$

A key additional feature is that agents face some borrowing constraint, which is set to be 0,

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<sup>10</sup>The non-annualized case is shown in appendix A.3.

Figure 3: Equilibrium under a Fall in the Markup: 2. OLG



and this limits the amount they may borrow each period:

$$A_{i,t+1} \geq 0$$

This problem can be solved by value function iteration. Ultimately, we get effectively the same solution as [Aiyagari \(1994\)](#)<sup>11</sup> The asset demand  $\bar{a}^d$  can then be computed for any equilibrium real rate  $\bar{r}$ .

$\bar{a}^d$  is plotted in figure 4 where a fall in the markup is considered (as in figure 1). A shift left in the asset supply due to a fall in the markup leads to a fall in relative assets and a fall in the equilibrium real rate.

The result of lowering the markup is different to the representative agent case because a fall in assets lowers the marginal utility in the next period by more than the current period since it

<sup>11</sup> It is necessary to make a minor change from Aiyagari which is to rewrite the problem using relative assets (this has no substantive impact upon the results however):

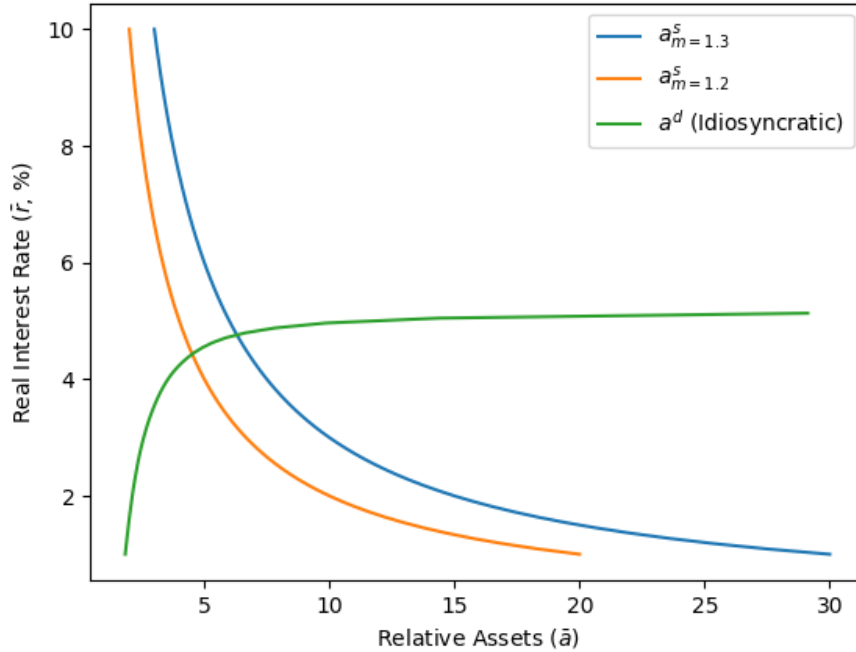
$$\max \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_{i,t}) \right]$$

s.t.

$$a_{i,t+1} = (1+r)a_t + \frac{L_{i,t}}{L_t} - c_{i,t}$$

$$a_{i,t+1} \geq 0$$

Figure 4: Equilibrium under a Fall in the Markup: 3. Idiosyncratic Labor



means that more agents will face a binding borrowing constraint in the next period. This means that agents want to save more. In turn, this raises the price of assets and lowers their real return in equilibrium (the equilibrium real rate).

The degree to which a shift left in assets lowers the equilibrium real rate depends upon whether many agents are close the borrowing constraints. When the level of assets is high (low), a fall in assets will increase a little (lot) the number of agents affected by the borrowing constraint so it will raise the demand for savings a little (lot) and thus lower the equilibrium real rate a little (lot). This can be seen in figure 4 and is reflected graphically in the steeper relative asset demand when relative assets are low.

### 3 Model

I now introduce the full model which is used to assess the importance of the channel and to conduct welfare analysis.

#### 3.1 Households

I start by describing the general overlapping generations framework. Each agent lives for  $M$  periods. Agents born in different periods overlap. An agent is denoted by its age in periods so an agent born  $i$  periods ago is denoted  $i$ . Therefore, the  $M$  cohorts in any given period are denoted

$0, \dots, M - 1$ . Each period: new agents are born (cohort 0), the oldest agents from the previous period (cohort  $M - 1$  at time  $t - 1$ ) have died and all other generations mature from cohort  $i$  to  $i + 1$ .

The population of the cohort born at time  $t$  is defined as  $N_t$ . The total population is defined as  $\mathbb{N}_t$  and thus  $\mathbb{N}_t = \sum_{i=0}^{M-1} N_{t-i}$ . It is assumed that the population grows at a constant rate of  $n$  so that  $N_{t+1} = (1 + n)N_t$ . Thus, the total population also grows by  $1 + n$  each year.

An agent of cohort  $i$  at time  $t$  has a budget constraint given by equation 20. An agent of cohort  $i$  consumes  $C_{i,t}$  at time  $t$ . An agent of cohort  $i$  works for  $L_{i,t}$ .  $W_t$  is the real wage paid at time  $t$  for each unit of work. An agent can invest in bonds, capital or shares in firms.  $B_{i,t}, K_{i,t}$  are respectively the bonds and capital held by agents of cohort  $i$  at the start of period  $t$  (so they were chosen at  $t - 1$  when that agent was cohort  $i - 1$ ). The bond is in nominal terms and pays interest rate  $i_{t-1}$  at time  $t$  (denoted with a  $t - 1$  since the nominal interest rate is chosen at  $t - 1$ ). Capital is in real terms and agents get a real return of  $r_t$  from selling their capital to the firm at time  $t$ .  $\tilde{\omega}_{i,t}$  is the number of shares of the composite firm that agent  $i$  owns at the start of time  $t$ . The total number of shares issued is 1 so  $\tilde{\omega}_{i,t}$  also represents the proportion of the firm owned by an agent of cohort  $i$  at time  $t$ . The price of a share is  $\tilde{Z}_t$  and it pays out a proportional amount of the firm's total profits  $\tilde{\Omega}_t$  each period. Assume the agent starts with zero assets so  $K_{i,0} = B_{i,0} = \tilde{\omega}_{i,0} = 0$ . For ease of notation, real and gross interest rates are also defined  $R_t = 1 + r_t, I_t = 1 + i_t$ .

$$C_{i,t} + \frac{B_{i+1,t+1}}{P_t} + K_{i+1,t+1} + \tilde{Z}_t \tilde{\omega}_{i+1,t+1} \leq W_t L_{i,t} + I_{t-1} \frac{B_{i,t}}{P_t} + R_t K_{i,t} + (\tilde{\Omega}_t + \tilde{Z}_t) \tilde{\omega}_{i,t} \quad (20)$$

The agent's lifetime utility function when they are in cohort  $k$  is given by equation 21. CRRA utility is used (equation 22). Both endogenous and exogenous labor are allowed for. In the exogenous labor case, the labor supply is fixed by each cohort so that  $L_{i,t} = L_i \forall t$  and the disutility of labor term  $v(L_{i,t})$  does not appear in the utility function. In the endogenous labor case, the disutility of labor is given by equation 23 where  $\eta$  is the elasticity of labor supply. Bonds are also allowed to have additional utility to the consumer. This is not key to the analysis and is only used to easily adjust the real interest on bonds when the optimal inflation target is considered in section 6.  $u_b$  (equation 24) is set so that the utility on bonds simplifies to give a fixed wedge between the steady state real interest rate on bonds and the steady state real interest rate on other assets (this is  $\xi$  in equation 25).<sup>12</sup>

$$\mathbb{E}_t \left[ \sum_{i=k}^{M-1} \beta^{i-k} \left[ u(C_{i,t}) + u_b \left( \frac{B_{i,t}}{P_t} \right) - v_i(L_{i,t}) \right] \right] \quad (21)$$

<sup>12</sup> $u_b$  (equation 24) is set so that the wedge simplifies easily in the Fisher equation.  $\xi$  is some constant utility from bonds. If  $\xi = 0$ , then the standard case without utility on bonds applies. Assuming  $\xi > 0$  then the utility from bonds depends upon the nominal return from bonds  $I_{t-1} \frac{B_t}{P_t}$  and the marginal utility of consumption. It is assumed that the agent does not take into account how changing their consumption will affect the marginal utility from safe bonds so that the condition simplifies easily. This is why  $\bar{C}_{i,t}$  is denoted with a bar.



where:

$$u(C) = \frac{C^{1-\gamma}}{1-\gamma} \quad (22)$$

$$v_i(L_{i,t}) = \frac{1}{1+\eta} x_i L_{i,t}^{1+\eta} \quad (23)$$

$$u_b\left(\frac{B_{i,t}}{P_t}\right) = \xi I_{t-1} \frac{B_{i,t}}{P_t} u'(\bar{C}_{i,t}) \quad (24)$$

Therefore, an agent of age  $k$  faces the following problem:

$$\max_{\{C_{i,t+i}, B_{i+1,t+i+1}, K_{i+1,t+i+1}, \tilde{\omega}_{i+1,t+i+1}\}_{i=k}^{M-1}} \mathbb{E}_t \left[ \sum_{i=k}^{M-1} \beta^{i-k} [u(C_{i,t}) + u_b\left(\frac{B_{i,t}}{P_t}\right) - v(L_{i,t})] \right]$$

s.t.  $\forall i \in k, \dots, M-1$ :

$$\begin{aligned} C_{i,t+i} + \frac{B_{i+1,t+i+1}}{P_{t+i}} + K_{i+1,t+i+1} + \tilde{Z}_{t+i} \tilde{\omega}_{i+1,t+i+1} \\ \leq W_{t+i} L_{i,t+i} + I_{t-1} \frac{B_{i,t+i}}{P_{t+i}} + R_{t+i} K_{i,t+i} + (\tilde{\Omega}_{t+i} + \tilde{Z}_{t+i}) \tilde{\omega}_{i,t+i} \end{aligned}$$

$$B_{M,t+M-k}, K_{M,t+M-k}, \omega_{M,t+M-k} \geq 0$$

First-order conditions are applied. This yields arbitrage conditions on bonds (equation 25), capital (equation 26) and shares (equation 27). Note that gross inflation is defined in the usual way ( $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$ ). Also observe that the only impact of the utility on bonds (equation 24) is to add the constant wedge ( $\xi$ ) into equation 25.  $\forall i \in 0, \dots, M-2$ :

$$u'(C_{i,t}) = \beta \mathbb{E}_t [u'(C_{i+1,t+1}) \frac{I_t}{\Pi_{t+1}} (1 + \xi)] \quad (25)$$

$$u'(C_{i,t}) = \beta \mathbb{E}_t [R_{t+1} u'(C_{i+1,t+1})] \quad (26)$$

$$\tilde{Z}_t u'(C_{i,t}) = \beta \mathbb{E}_t [u'(C_{i+1,t+1}) (\tilde{\Omega}_{t+1} + \tilde{Z}_{t+1})] \quad (27)$$

With endogenous labor, it is derived  $\forall i \in 0, \dots, M-1$ :

$$W_t u'(C_{i,t}) = v'(L_{i,t}) \quad (28)$$

To make the model tractible, the share holdings by generation are rewritten in per capita terms. Define  $\omega_{i,t} = \mathbb{N}_t \tilde{\omega}_{i,t}$  so that  $\omega_{i,t}$  represents the proportional per capita holdings of an agent of cohort  $i$  at time  $t$  of firm shares rather than the aggregate holdings of cohort  $i$  at  $t$ . Then define  $Z_t$  to be the price of a per capita share in firms i.e.  $Z_t = \frac{\tilde{Z}_t}{\mathbb{N}_t}$  and  $\Omega_t$  to be the profits paid by a per capita share in firms i.e.  $\Omega_t = \frac{\tilde{\Omega}_t}{\mathbb{N}_t}$ . Equations 20 and 27 become respectively:

$$Z_t u'(C_{i,t}) = \beta \mathbb{E}_t [u'(C_{i+1,t+1}) (1+n) (\Omega_{t+1} + Z_{t+1})] \quad (29)$$

$$C_{i,t} + \frac{B_{i+1,t+1}}{P_t} + K_{i+1,t+1} + \frac{Z_t \omega_{i+1,t+1}}{1+n} \leq W_t L_{i,t} + I_{t-1} \frac{B_{i,t}}{P_t} + (1+r_t)K_{i,t} + (\Omega_t + Z_t)\omega_{i,t} \quad (30)$$

All conditions needed to study the long-run equilibrium have been derived. However, to consider the impact of shocks, it is necessary to make some further adjustments to the household conditions.

Define the amount that agents of cohort  $i$  have available at the start of  $t$  from savings they made in  $t-1$  as  $T_{i,t}$  (equation 32). Define the amount that agents of cohort  $i$  save at  $t$  for  $t+1$  as  $S_{i+1,t}^p$  (equation 31).<sup>13 14</sup>

$$S_{i+1,t}^p = \frac{B_{i+1,t+1}}{P_t} + K_{i+1,t+1} + \frac{Z_t \omega_{i+1,t+1}}{1+n} \quad (31)$$

$$T_{i,t} = I_{t-1} \frac{B_{i,t}}{P_t} + R_t K_{i,t} + (\Omega_t + Z_t)\omega_{i,t} \quad (32)$$

Observe that the budget constraint (equation 30) can be rewritten as:

$$C_{i,t} + S_{i+1,t}^p \leq W_t L_{i,t} + T_{i,t} \quad (33)$$

Define  $T_t$  to be the per capita aggregate savings held at the start of a period  $t$  from savings made at  $t-1$  (equation 34). This can be computed this by summing the population-weighted savings held by each cohort ( $\sum_{i=0}^{M-1} N_{t-i} T_{i,t}$ ) divided by the total population ( $\mathbb{N}_t$ ). It is then possible to simplify this slight by rewriting the population structure  $N_{t-i}, \mathbb{N}_t$  in terms of  $n$ . These steps are shown in equation 34.

$$T_t = \frac{\sum_{i=0}^{M-1} N_{t-i} T_{i,t}}{\mathbb{N}_t} = \frac{\sum_{i=0}^{M-1} N_{t-i} T_{i,t}}{\sum_{i=0}^{M-1} N_{t-i}} = \frac{\sum_{i=0}^{M-1} N_t \frac{1}{(1+n)^i} T_{i,t}}{\sum_{i=0}^{M-1} N_t \frac{1}{(1+n)^i}} = \frac{\sum_{i=0}^{M-1} \frac{1}{(1+n)^i} T_{i,t}}{\sum_{i=0}^{M-1} \frac{1}{(1+n)^i}} \quad (34)$$

Using the simplified per capita definition, additional variables are defined:  $S_t^p$  is the per capita savings made at  $t$  for  $t+1$  (equation 35);  $B_t$  is the per capita bonds held at the start of period  $t$ ;  $K_t$  is the per capita capital held at the start of period  $t$  (equation 37);  $\omega_t$  is the per capita holdings of shares at the start of period  $t$  (equation 38).

$$S_t^p = \frac{\sum_{i=0}^{M-1} \frac{1}{(1+n)^i} S_{i+1,t}^p}{\sum_{i=0}^{M-1} \frac{1}{(1+n)^i}} \quad (35)$$

$$B_t = \frac{\sum_{i=0}^{M-1} \frac{1}{(1+n)^i} B_{i,t}}{\sum_{i=0}^{M-1} \frac{1}{(1+n)^i}} \quad (36)$$

<sup>13</sup>A superscript  $p$  is used to represent the fact that these are savings held by agents at the end of  $t$  (which is different to how capital and  $T_t$  are defined).

<sup>14</sup>Note that  $T_{0,t}, S_{M,t}^p = 0$  which makes sense since agents don't hold assets when they are born or when they are about to die.

$$K_t = \frac{\sum_{i=0}^{M-1} \frac{1}{(1+n)^i} K_{i,t}}{\sum_{i=0}^{M-1} \frac{1}{(1+n)^i}} \quad (37)$$

$$\omega_t = \frac{\sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \omega_{i,t}}{\sum_{i=0}^{M-1} \frac{1}{(1+n)^i}} \quad (38)$$

Next, recall that the total holdings of shares ( $\tilde{\omega}_{i,t}$ ) in a firm must sum to 1 i.e.  $\sum_{i=0}^{M-1} N_{t-i} \tilde{\omega}_{i,t} = 1$ . Applying the definition of  $\omega_{i,t} = \tilde{\omega}_{i,t} N_t$  implies that the aggregate per capita holdings of shares in a firm  $\omega_t$  must also always equal 1. This is shown formulaically in equation 39.

$$\omega_t = \frac{\sum_{i=0}^{M-1} N_{t-i} \omega_{i,t}}{N_t} = \frac{\sum_{i=0}^{M-1} N_{t-i} \tilde{\omega}_{i,t} N_t}{N_t} = \sum_{i=0}^{M-1} N_{t-i} \tilde{\omega}_{i,t} = 1 \quad (39)$$

Inputting equation 32 into equation 34 and then applying equations 36 to 39 yields equation 40. Equation 40 just states that the total assets held at  $t$  equal the total return on bonds, capital and shares. Similarly, inputting equation 31 into equation 35 and then applying equations 36 to 39 yields equation 41.<sup>15</sup> Equation 41 just states that the total savings made at  $t$  equals next period capital and bonds plus the value of shares purchased.

$$T_t = I_{t-1} \frac{B_t}{P_t} + R_t K_t + \Omega_t + Z_t \quad (40)$$

$$S_t^p = (1+n) \frac{B_{t+1}}{P_t} + (1+n) K_{t+1} + Z_t \quad (41)$$

Define the share of savings of each cohort to be  $s_{i,t-1}^p = \frac{S_{i,t-1}^p}{S_t^p}$ . The definition of equation 35 can then be applied to show that the per capita value of  $s_{i,t}^p$  equals 1 in equation 42.

$$1 = \frac{\sum_{i=0}^{M-1} \frac{1}{(1+n)^i} s_{i,t-1}^p}{\sum_{i=0}^{M-1} \frac{1}{(1+n)^i}} \quad (42)$$

Next, set  $T_{i,t} = s_{i,t-1}^p T_t$ . This implies that the amount of total assets that a cohort holds at time  $t$  is proportional to the share of saving they did at time  $t-1$ . Equation 33 then can be rewritten as:

<sup>15</sup>To derive equation 41, the following steps can be made for bonds, capital and shares:

$$\frac{\sum_{i=0}^{M-1} N_{t-i} K_{i+1,t+1}}{N_t} = \frac{\sum_{i=1}^M N_{t-i} K_{i,t+1}}{N_t} = \frac{\sum_{i=0}^{M-1} N_{t-i} K_{i,t+1}}{N_t} = (1+n) \frac{\sum_{i=0}^{M-1} N_{t-i} K_{i,t+1}}{N_{t+1}} = (1+n) K_{t+1}$$

The first equality is just an adjustment of the summation index. The second equality uses  $K_{0,t+1} = K_{M,t+1} = 0$  to adjust the summation begin and start points. The third equality adjusts  $N_t$ . The fourth equality is just a definition.

$$C_{i,t} + s_{i+1,t}^p S_{t+1} = W_t L_{i,t} + s_{i,t-1}^p T_t \quad (43)$$

### 3.2 Firms

**Final Goods Firm** There is a single competitive final goods firm which aggregates goods in different industries to produce a final good. There are  $J$  industries in total, denoted  $1, \dots, J$ . The final goods firm has CES production and each industry has a weight  $a_j$  in production:

$$\left( \sum_{j=1}^J a_j^{\frac{1}{\sigma_2}} Y_{j,t}^{\frac{\sigma_2-1}{\sigma_2}} dj \right)^{\frac{\sigma_2}{\sigma_2-1}} = Y_t$$

Therefore, the final goods firm has the usual CES demand (taking into account industry weights) for each industry good given by equation 44. The price aggregator also takes the usual form given by equation 45. Note that weights  $a_j$  need to be added for each industry.

$$Y_{j,t} = a_j \left( \frac{P_{j,t}}{P_t} \right)^{-\sigma_2} Y_t \quad (44)$$

$$P_t = \left( \sum_{j=1}^J a_j P_{j,t}^{1-\sigma_2} dj \right)^{\frac{1}{1-\sigma_2}} \quad (45)$$

**Industry Aggregator** I allow for different industries with different weights and degrees of price rigidity. This is done for two reasons. The primary reason is that allowing for different degrees of price rigidities increases the degree of monetary non-neutrality which is otherwise unrealistically low. See [Carvalho \(2006\)](#) for a detailed discussion. It is also more realistic to allow for different industries with different degrees of price rigidity.

A perfectly competitive firm of firm  $j$  aggregates all the intermediate goods in that industry to produce the good for sector  $j$ . The sector firm has the following production function:

$$Y_{j,t} = \left( \int_0^1 Y_{i,j,t}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

---

<sup>16</sup> We will consider a first order perturbation. A first order perturbation means that agents only care about the expected return and do not care about risk. Therefore, all cohorts are indifferent between holding equivalently valued capital, bonds or shares since they all give a real expected return of  $\mathbb{E}_t[R_{t+1}]$ . This would also hold without a first order perturbation in a purely deterministic model without any risk. Thus, although the savings of each cohort is known, how savings are comprised is not known i.e.  $\frac{B_{i+1,t+1}}{P_t} + K_{i+1,t+1} + Z_t \omega_{i+1,t+i+1}$  is known but not  $\frac{B_{i+1,t+1}}{P_t}$  or  $K_{i+1,t+1}$  or  $\omega_{i+1,t+i+1}$ .

This does not matter when there is no risk since these assets will always return the same by arbitrage. However, it does matter when there is risk since if profits fall, agents who hold relatively more shares suffer. The model is kept simple by effectively assuming that agents hold proportional amounts of bonds, capital and shares. This avoids complications where shocks lead to unexpected redistribution.

Therefore, the industry aggregator has the usual CES demand for each intermediate good given by equation 46. The price aggregator also takes the usual form given by equation 47.

$$Y_{i,j,t} = Y_{j,t} \left( \frac{P_{i,j,t}}{P_{j,t}} \right)^{-\sigma} \quad (46)$$

$$P_{j,t} = \left( \int_0^1 P_{i,j,t}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \quad (47)$$

**Intermediate Goods Firms Cost Minimisation** The output of an intermediate firm  $i$  in industry  $j$  at time  $t$  is given by equation 48. Intermediate firms have Cobb Douglas production over capital ( $K_{i,j,t}$ ) and labor ( $L_{i,j,t}$ ). Productivity is denoted  $A_t$ .

$$Y_{i,j,t} = A_t K_{i,j,t}^\alpha L_{i,j,t}^{1-\alpha} \quad (48)$$

Real profits of an intermediate firm  $\Omega_{i,j,t}$  in a single period are given by equation 49. They rent capital from consumers at real rate  $r_t$ . They also have to refund consumers for the depreciation  $\delta$  in capital. They pay workers a real wage  $W_t$  for each unit of labor. A tax (surplus)  $\tau$  on renting capital and labor is introduced. In equilibrium, the lump sum transfer is set so that each period the amount transferred to the firm equals the tax (subsidy) it paid (received) on renting capital and labor (so the only impact of the tax is to adjust the cost of production for the firm). Firms do not observe that the tax will be transferred back to them hence why the transfer is shown in curly brackets in equation 49<sup>17</sup>

$$\Omega_{i,j,t} = \frac{P_{i,j,t} Y_{i,j,t}}{P_t} - (1 + \tau)((r_t + \delta)K_{i,j,t} + W_t L_{i,j,t}) + \{\tau((r_t + \delta)K_{i,j,t} + W_t L_{i,j,t})\} \quad (49)$$

Intermediate firms minimise costs in the standard manner, which requires that equations 50 and 51 hold.  $MC_t$  represents the marginal cost of the firm before tax. The problem is shown in detail in appendix B.1.1

$$MC_t = \frac{r_t + \delta}{\alpha A_t K_t^{\alpha-1} L_t^{1-\alpha}} \quad (50)$$

$$MC_t = \frac{W_t}{(1 - \alpha) A_t K_t^\alpha L_t^{-\alpha}} \quad (51)$$

Output and profits (equations 48 and 49) can be aggregated to get equations 52 and 53. It is also possible to write profits in the more usual form given in equation 54. These steps are discussed in appendix B.1.2

$$Y_t \nu_t = A_t K_t^\alpha L_t^{1-\alpha} \quad (52)$$

$$\Omega_t = Y_t - Y_t MC_t \nu_t \quad (53)$$

$$\Omega_t = Y_t - (r_t + \delta)K_t - W_t L_t \quad (54)$$

<sup>17</sup>I introduce the tax so the equilibrium real rate can be set to take a particular value.

As part of the aggregation of output and profits it is necessary to define a price dispersion variable  $\nu_t$  (defined in equation 55) which in turn aggregates the price dispersion of individual industries  $\nu_{j,t}$  (defined in equation 56).

$$\nu_t = \sum_{j=1}^J a_j \left( \frac{P_{j,t}}{P_t} \right)^{-\sigma_2} \nu_{j,t} dj \quad (55)$$

$$\nu_{j,t} = \int_0^1 \left( \frac{P_{i,j,t}}{P_{j,t}} \right)^{-\sigma} di \quad (56)$$

**Rewriting Cost Minimisation Conditions in terms of the Markup** The average markup  $m_t$  is defined to be the inverse of the marginal cost of producing one final good i.e. equation 57.<sup>18</sup> Profits (equation 53) are written in terms of the markup in equation 58. Using equation 52 as well, equations 50 and 51 can be rewritten in terms of  $m_t$  as equations 59 and 60

$$m_t = \frac{1}{MC_t \nu_t} \quad (57)$$

$$\Omega_t = \left(1 - \frac{1}{m_t}\right) Y_t \quad (58)$$

$$\frac{\alpha}{m_t} = \frac{(r_t + \delta) K_t}{Y_t} \quad (59)$$

$$\frac{1 - \alpha}{m_t} = \frac{W_t L_t}{Y_t} \quad (60)$$

**Intermediate Firm Profit Maximisation** Firms in each industry  $j$  have a  $\lambda_j$  probability of updating their price each period. When they do get to change their price, firms maximise equation 61 subject to the demand for their good from the industry aggregator firm (equation 46). Firms discount future real profits by a fixed amount  $\frac{\beta_f}{R}$ . The  $\frac{1}{R}$  represents the risk-free discount of the future. Firms are allowed to discount by an additional  $\beta_f$ . Therefore, firms maximise equation 61 subject to the demand from industry aggregator firms (equation 46).

$$\max_{P_{j,t}^* Y_{i,j,t}} \sum_{k=0}^{\infty} \left( \frac{\beta_f}{R} \right)^k (1 - \lambda_j)^k \left[ \frac{P_{j,t}^* Y_{i,j,t+k}}{P_{t+k}} - (1 + \tau) MC_{j,t+k} Y_{i,j,t+k} \right] \quad (61)$$

**Rewriting Price Evolution Equations** Equation 56 can be rewritten as equation 62. Equation 47 can be rewritten as equation 63. There is a relationship between inflation in an industry and the relative price in that industry that holds by definition and is shown in equation 64. And

<sup>18</sup>This includes the degree of price dispersion because as the price dispersion increases, demand for intermediate goods with cheaper prices rises even though these goods contribute less to making a final good than less used goods with more expensive prices. Thus, more intermediate goods must be used to produce a final good than in the case where there is no price dispersion.

equation 45 can be rewritten as equation 65. These steps are discussed in appendix B.1.3.

$$\nu_{j,t} = \lambda_j \left( \frac{P_{j,t}^*}{P_{j,t}} \right)^{-\sigma} + (1 - \lambda_j) \nu_{j,t-1} \Pi_{j,t}^\sigma \quad (62)$$

$$1 = \lambda_j \left( \frac{P_{j,t}^*}{P_{j,t}} \right)^{1-\sigma} + (1 - \lambda_j) \Pi_{j,t}^{\sigma-1} \quad (63)$$

$$\Pi_{j,t} = \frac{P_{j,t}}{P_t} \frac{P_t}{P_{t-1}} \frac{P_{t-1}}{P_{j,t-1}} \quad (64)$$

$$1 = \sum_{j=1}^J a_j \left( \frac{P_{j,t}}{P_t} \right)^{1-\sigma_2} \quad (65)$$

**Intermediate Firm Profit Maximisation Solution** The solution to equation 61 can be rewritten as the first-order condition (equation 66) plus two auxilliary equations (equations 67 and 68). The derivation is discussed in appendix B.1.4.

$$U_{j,t} \frac{P_{j,t}^*}{P_{j,t}} \frac{P_{j,t}}{P_t} - V_{j,t} = 0 \quad (66)$$

$$U_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\sigma_2} Y_t + \mathbb{E}_t \left[ \frac{\beta f}{R} (1 - \lambda_j) \Pi_{j,t+1}^\sigma \Pi_{t+1}^{-1} U_{j,t+1} \right] \quad (67)$$

$$V_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\sigma_2} Y_t \frac{\sigma}{\sigma - 1} (1 + \tau) MC_t + \mathbb{E}_t \left[ \frac{\beta f}{R} (1 - \lambda_j) \Pi_{j,t+1}^\sigma V_{j,t+1} \right] \quad (68)$$

### 3.3 Monetary and Fiscal Policy

When investigating the long-run equilibrium, a monetary rule does not need to be specified (since we're just computing the steady state). In this case, just note that the central bank holds inflation at some target  $\pi^*$ . However, a monetary rule is needed when investigating the equilibrium with shocks. A similar monetary rule to Coibion et al. (2012) is used which is given in equation 69.<sup>19</sup>

$$I_t = \max \left\{ I_{t-1}^{\rho_{i1}} I_{t-2}^{\rho_{i2}} \left( \frac{\Pi_t}{\bar{\Pi}} \right)^{\phi_\pi} \left( \frac{Y_t}{\bar{Y}} \right)^{\phi_y} \right\}^{1-\rho_{i1}-\rho_{i2}}, 0 \quad (69)$$

The government is assumed to have no debt/savings:

$$B_t = 0$$

### 3.4 Other Conditions

In the main model,  $A_t = 1$ .

<sup>19</sup>The one change is that the interest rate responds to the difference from output from its steady state rather than its natural level.



Total labor is just the population-weighted sum of labor given by equation 70. In the exogenous labor case,  $L_t$  is effectively fixed.

$$L_t = \frac{\sum_{i=0}^{M-1} \left(\frac{1}{1+n}\right)^i L_{i,t}}{\sum_{i=0}^{M-1} \left(\frac{1}{1+n}\right)^i} \quad (70)$$

## 4 Model Solution and Calibration

In this section, I discuss how the conditions derived in section 3 can be used to do policy analysis.

### 4.1 Full Conditions

In this subsection, the conditions derived in section 3 are summarized.

The household's problem is summarized by  $2M+4$  conditions:  $M-1$  Euler condition(s) (equation 26), two arbitrage conditions (equations 25 and 29), the sum of savings shares (equation 42), the amounts of savings and assets (equations 40 and 41) and  $M$  simplified budget constraints (equation 43).

The firm's cost minimisation problem is summarized by 4 conditions: the cost minimisation conditions (equations 50 and 51), the definition of output (equation 48) the definition of profits (equation 49).

For the firm's pricing problem: There is a condition for each industry for equations 62 to 64 and 66 to 68. There are also two overall conditions (equations 55 and 65). In total, the firm's pricing problem is summarized by  $6J+2$  conditions.

There is one condition from monetary policy (equation 69) and one equilibrium condition (equation 70).

In total, there are  $2M+6J+12$  conditions. These correspond to the following variables:

$$\{C_{i,t}\}_{i=0}^{M-1}, \{s_{i,t}\}_{i=1}^{M-1}, I_t, \Pi_t, R_t, W_t, MC_t, K_t, L_t, Y_t, Z_t, \Omega_t, S_t, T_t, \nu_t, \left\{ \frac{P_{j,t}}{P_t}, \frac{P_{j,t}^*}{P_{j,t}}, \Pi_{j,t}, \nu_{j,t}, U_{j,t}, V_{j,t} \right\}_{j=1}^J$$

### 4.2 Steady State

In this subsection, the long-run equilibrium (the steady state) of the model is computed.

I solve for the steady state in a similar manner to section 2. The relative asset demand and relative asset supply are computed and then I find equilibria where they intersect. Relative assets  $a_t$  are defined to be total savings held by agents at the end of a period ( $S_t^p$ ) divided by labor income ( $W_t L_t$ ). This is shown in equation 71. The reason relative assets are used is because then asset demand doesn't depend upon the wage which makes the model easier to solve. In graphs, references are made to "annualized assets" which are just assets divided by annualized labor income rather than labor income for one period.

$$a_t = \frac{S_t^p}{W_t L_t} \quad (71)$$

The solution is broken into three parts. Firstly, the markup  $\bar{m}$  is solved for given the inflation target; this is explained in appendix C.1. Secondly, the supply of relative assets  $\bar{a}^s$  is solved for given the markup; this is explained in appendix C.2. Thirdly, the demand for relative assets  $\bar{a}^d$  is solved for; this is explained in appendix C.3. It is then possible to find the steady state by looking for points where the supply and demand for relative assets intersect.

I demonstrate that the equilibrium must exist, is dynamically efficient and satisfies  $\bar{R} > 1 + n$  in appendix D.

### 4.3 Shocks and Log-linearized Conditions

In this subsection, shocks are incorporated into the log-linearised versions of the full set of model conditions found in section 4.1.

Similar shocks to Coibion et al. (2012) are incorporated into the model.<sup>20</sup> The shocks are to technology, the risk premium, the Phillips Curve and the nominal interest rate and are denoted as respectively  $\epsilon_{a,t}, \epsilon_{q,t}, \epsilon_{m,t}, \epsilon_{i,t}$  with standard deviations of the shocks respectively given as  $\sigma_a, \sigma_q, \sigma_m, \sigma_i$ . The productivity, the risk premium and the Phillips Curve shocks are AR(1) processes:

$$\hat{A}_t = \rho_a \hat{A}_{t-1} + \epsilon_{a,t}$$

$$\hat{q}_t = \rho_q \hat{q}_{t-1} + \epsilon_{q,t}$$

$$\hat{m}_t = \rho_m \hat{m}_{t-1} + \epsilon_{m,t}$$

Denote the log linearisation of  $X$  around steady state as  $\hat{X}$ :

$$\hat{X}_t = \log(X_t) - \log(\bar{X})$$

Household conditions (equations 25, 26, 29 and 40 to 43) are log-linearized to yield:

$$\mathbb{E}_t[\hat{C}_{i+1,t+1}] = \frac{1}{\gamma} (\mathbb{E}_t[\hat{R}_{t+1}] + \hat{q}_t) + \hat{C}_{i,t}$$

$$\mathbb{E}_t[\hat{R}_{t+1}] = \hat{I}_t - \mathbb{E}_t[\hat{\Pi}_{t+1}]$$

$$\hat{Z}_t + \mathbb{E}_t[\hat{R}_{t+1}] = \frac{\bar{\Omega}}{\bar{\Omega} + \bar{Z}} \mathbb{E}_t[\hat{\Omega}_{t+1}] + \frac{\bar{Z}}{\bar{\Omega} + \bar{Z}} \mathbb{E}_t[\hat{Z}_{t+1}]$$

$$0 = \frac{\sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \bar{s}_i \hat{s}_{i,t}}{\sum_{i=0}^{M-1} \frac{1}{(1+n)^i}}$$

$$\bar{T}\hat{T}_t = \bar{R} \frac{\bar{B}}{\bar{P}} \left( \widehat{\frac{B_t}{P_{t-1}}} \right) + \bar{R}\bar{K}(\hat{R}_t + \hat{K}_t) + \bar{\Omega}\hat{\Omega}_t + \bar{Z}\hat{Z}_t$$

<sup>20</sup>The differences are that this model does not have a government sector so it does not have no government shocks and productivity growth does not follow some trend here.

$$\bar{S}^p \hat{S}_t^p = \frac{\bar{B}}{\bar{P}} \left( \widehat{\frac{B_{t+1}}{P_t}} \right) + \bar{K} \hat{K}_{t+1} + \bar{Z} \hat{Z}_t$$

$$\bar{C}_i \hat{C}_{i,t} + \bar{s}_{i+1}^p \bar{S}^p [\hat{s}_{i+1,t}^p + \hat{S}_{t+1}^p] = \bar{W} \bar{L}_i \hat{W}_t + \bar{s}_i^p \bar{T} [\hat{s}_{i,t-1}^p + \hat{T}_t]$$

Firm cost minimization conditions (equations 50 to 53) are log-linearized to yield:

$$\hat{M}C_t = \hat{r}_t - \hat{A}_t + (1 - \alpha) \hat{K}_t$$

$$\hat{M}C_t = \hat{W}_t - \hat{A}_t - \alpha \hat{K}_t$$

$$\hat{Y}_t + \hat{v}_t = \hat{A}_t + \alpha \hat{K}_t$$

$$\hat{\Omega}_t = \hat{Y}_t - \frac{\bar{M}C\bar{v}}{1 - \bar{M}C\bar{v}} (\hat{M}C_t + \hat{v}_t)$$

Profit maximization conditions (equations 55 and 62 to 68) are log-linearized to yield:

$$\bar{v}_j \hat{v}_{j,t} = -\sigma \lambda_j \left( \frac{P_j^*}{P_j} \right)^{-\sigma} \left( \widehat{\frac{P_{j,t}^*}{P_{j,t}}} \right) + (1 - \lambda_j) \bar{v}_j \bar{\Pi}^\sigma (\hat{v}_{j,t-1} + \sigma \hat{\Pi}_{j,t})$$

$$0 = (1 - \sigma) \bar{\lambda}_j \left( \frac{P_j^*}{P_j} \right)^{1-\sigma} \left( \widehat{\frac{P_{j,t}^*}{P_{j,t}}} \right) + (\sigma - 1)(1 - \lambda_j) \bar{\Pi}^{\sigma-1} \hat{\Pi}_{j,t}$$

$$\hat{\Pi}_{j,t} = \left( \widehat{\frac{P_{j,t}}{P_t}} \right) + \hat{\Pi}_t - \left( \widehat{\frac{P_{j,t-1}}{P_{t-1}}} \right)$$

$$\hat{U}_{j,t} + \left( \widehat{\frac{P_{j,t}^*}{P_{j,t}}} \right) + \left( \widehat{\frac{P_{j,t}}{P_t}} \right) = \hat{V}_{j,t}$$

$$\bar{U}_j \hat{U}_{j,t} = \left( \frac{\bar{P}_j}{\bar{P}} \right)^{-\sigma_2} \bar{Y} \left( -\sigma_2 \left( \widehat{\frac{P_{j,t}}{P_t}} \right) + \hat{Y}_t \right) + \frac{\beta_f}{\bar{R}} (1 - \lambda_j) \bar{\Pi}^{\sigma-1} \bar{U} (\sigma \hat{\Pi}_{j,t+1} - \hat{\Pi}_{t+1} + \hat{U}_{j,t+1})$$

$$\bar{V}_j \hat{V}_{j,t} = \left( \frac{\bar{P}_j}{\bar{P}} \right)^{-\sigma_2} \bar{Y} \frac{\sigma}{\sigma - 1} (1 + \tau) \bar{M}C \left( -\sigma_2 \left( \widehat{\frac{P_{j,t}}{P_t}} \right) + \hat{Y}_t + \hat{M}C_t + \hat{m}_t \right) + \frac{\beta_f}{\bar{R}} (1 - \lambda_j) \bar{\Pi}^\sigma \bar{V} (\sigma \hat{\Pi}_{j,t+1} + \hat{V}_{j,t+1})$$

$$0 = \sum_{j=1}^J (1 - \sigma_2) a_j \left( \frac{\bar{P}_j}{\bar{P}} \right)^{1-\sigma_2} \left( \widehat{\frac{P_{j,t}}{P_t}} \right)$$

$$\bar{v} \hat{v}_t = - \sum_{j=1}^J \sigma_2 a_j \left( \frac{\bar{P}_j}{\bar{P}} \right)^{-\sigma_2} \bar{v}_j \left( \left( \widehat{\frac{P_{j,t}}{P_t}} \right) + \hat{v}_{j,t} \right)$$

The monetary rule (equation 69) is log-linearized:

$$\hat{I}_t = \max\{\rho_{i1} \hat{I}_{t-1} + \rho_{i2} \hat{I}_{t-2} + (1 - \rho_{i1} - \rho_{i2})(\phi_\pi \hat{\Pi}_t + \phi_y \hat{Y}_t) + \epsilon_{i,t}, -\log(\bar{I})\} \quad (72)$$

Labor in equilibrium (equation 70) is log-linearized:

$$\hat{L}_t = \frac{\sum_{i=0}^{M-1} \left(\frac{1}{1+n}\right)^i \hat{L}_{i,t}}{\sum_{i=0}^{M-1} \left(\frac{1}{1+n}\right)^i}$$

#### 4.4 Policy Functions and Simulation

In this subsection, the method used to simulate the log-linearized model is discussed.

Firstly, note that a standard DSGE model has been derived. Therefore, a first-order linear perturbation of the conditions given in section 4.3 can be applied. However, any first-order perturbation will ignore the ZLB in equation 72.

To capture the impact of the ZLB, a similar method to [Guerrieri and Iacoviello \(2015\)](#) is used. The basic idea is that agent's choices at time  $t$  are solved for by finding what they would do without shocks in the future if there was no ZLB. If the central bank would set a nominal interest rate below zero in this case then the estimation is rerun with the central bank constrained in certain periods to be at the ZLB. This process continues until a situation where the central bank is not setting a nominal interest rate below zero is found. The exact algorithm used is as follows:

1. Set  $t$  to be the first period of the simulation.
2. Assume that at  $t + 100$ , without any further shocks, the ZLB will no longer bind.
3. Guess the following regime: The ZLB does not bind in any period from  $t$  to  $t + 99$ .
4. Solve backwards from  $t+99$  to  $t$  to get the policy functions for each period under the guessed regime.<sup>21</sup>
5. Using the policy functions computed in step 3, solve out for the path of the economy from  $t$  to  $t + 99$  in the case where there are no shocks from  $t + 1$  to  $t + 99$ .
6. Verify whether the nominal interest rate is always greater than or equal to zero in every period. If it is not:
  - (a) If this regime is such that the economy was never at the ZLB, set that the regime is now such that the ZLB binds at  $t$  but not in future periods. If this regime is such that the economy was at the ZLB until period  $t + s$ , set that the regime is now such that the economy is at the ZLB until  $t + s + 1$ .<sup>22</sup>
  - (b) Go back to step 3.
7. Take agent's choices at  $t$  to be the simulation values for  $t$ . Now set  $t$  to be the next period of the simulation and go back to step 2.

<sup>21</sup>The computations used to solve backwards from  $t + 99$  to  $t$  is the same as in [Guerrieri and Iacoviello \(2015\)](#).

<sup>22</sup>I have not encountered the situation where the ZLB binds from  $t$  to  $t + 99$ .

The difference between my method and [Guerrieri and Iacoviello \(2015\)](#) is in step 6. In my method, step 6 implies that if a central bank knows that it would hit the ZLB in the future without additional shocks, it will lower its nominal interest rate to zero in all preceding periods.<sup>23</sup> Guerrieri and Iacoviello’s algorithm implies that the ZLB should only bind in periods when the nominal interest rate would have been below zero according to the monetary rule. This means the ZLB can bind in nonconsecutive periods and can bind in the future even if it doesn’t bind today.

Guerrieri and Iacoviello warn that their method is a simple first pass and I find there are advantages to adapting their method to what I have described here. Firstly, since it is more general, the Guerrieri and Iacoviello method often does not converge in my model. Secondly, my method works more quickly. Thirdly, I think it is more realistic to assume that if a central bank knows that the economy is likely to hit the ZLB in the future, the central bank will lower nominal interest rates to zero immediately rather than waiting.

## 4.5 Calibration

**General Parameters** Each period is set to represent a quarter. Standard parameters are set as follows:  $\alpha = 0.3$ ,  $\beta = 0.98^{\frac{1}{4}}$ ,  $\delta = 0.1^{\frac{1}{4}}$ ,  $\gamma = 2$ .  $\xi$  is set to be 0 so there is no premium on safe bonds in the baseline calibration.

$M = 220$  is calibrated to capture each quarter of life of an adult between the ages of 24 and 78. The simulation begins at age 24 to avoid having to worry about how to capture college. Agents’ last year of life is 78 because the life expectancy of someone in the US is currently just under 79 years.

With exogenous labor supply,  $\bar{L}_i$  (hours worked by each age) is set to match the average hours worked of a person of that age in the American Time Use Survey between 2003 and 2016. With endogenous labor supply:  $x_i$  in the disutility of labor function (equation 23) is set so that when  $\beta\bar{R} = 1$  we have that  $\bar{L}_i$  matches the hours worked in the exogenous case. This is explained in appendix E.1.  $\eta$  is set to be 1 in the endogenous labor supply case.

The industry weights and frequencies of price adjustment are set to match regular non-sale prices in [Nakamura and Steinsson \(2008\)](#). The elasticity of substitution between varieties within industries ( $\sigma$ ) is set to be  $\sigma = 8$ . This is in between the lower and upper bounds of respective 6 and 10 used in [Carvalho et al. \(2016\)](#). The elasticity of substitution between industries ( $\sigma_2$ ) is set to 1 as in [Shamloo \(2010\)](#)<sup>24</sup>.

It is important that firm discounting is set correctly since it makes a difference for the size of the first part of the channel. The degree of firm discounting is based upon the Weighted Average Cost of Capital (WACC) which is the average a company is expected to pay to finance its assets.<sup>25</sup> [Jagannathan et al. \(2016\)](#) estimated that it was 8% in 2003 when the expected ten year rate on

<sup>23</sup>My algorithm could also be considered as a simpler version of [Andrade et al. \(2018\)](#) who also adapt the algorithm of [Guerrieri and Iacoviello \(2015\)](#). They allow for the ZLB to start binding at some period  $t_1 \geq t$  and then stop binding at some later period  $t_2 > t_1$ . I effectively set that  $t_1 = t$  always.

<sup>24</sup>I actually set it to be 1.001 otherwise I would have to rewrite the indices since 1 is a special limiting case.

<sup>25</sup>This is used since it is the cost to the firm of not obtaining funds earlier by setting a lower markup.

real bonds ( $r^e$ ) was 2.8p.p. [Graham and Harvey \(2011\)](#) estimated it was 10.0% in 2011Q1 when  $r^e$  was 2.2%. [Graham and Harvey \(2012\)](#) then estimated it was 9.3% in 2012Q2 when  $r^e = 1.3\%$ . From these three surveys, the average wedge between WACC and the expected real rate is 7p.p. Therefore, a firm discount of  $\beta_f = \left(\frac{1}{1.07}\right)^{\frac{1}{4}}$  is applied.

$\bar{r}$  is set to be 2.06% when  $\pi^* = 2\%$ . This matches the average real interest rate on treasury bills between 1995 and 2007.  $\tau$  (the tax on the labor and capital inputs) is calibrated to set  $\bar{r}$  at this level.

**Simulation Parameters** The parameters needed to solve for the long-run (steady state) equilibrium have been fully described. I now describe the parameters that are only needed for the simulation.

The same monetary rule parameters are used as in [Coibion et al. \(2012\)](#):  $\rho_{i,1} = 1.05$ ,  $\rho_{i,2} = -0.13$ ,  $\phi_\pi = 2.5$ ,  $\phi_y = 0.11$ .

Where possible, the shock parameters are set to be the same as in [Coibion et al. \(2012\)](#). Therefore:  $\sigma_a = 0.009$ ,  $\sigma_q = 0.0024$ ,  $\sigma_m = 0.0014$ ,  $\sigma_i = 0.0024$ ,  $\rho_q = 0.947$ ,  $\rho_m = 0.9$ . The persistence in productivity is set to be  $\rho_a = 0.9$ .

## 5 Impact of Raising the Inflation Target

In this section, I consider the impact of raising the annual inflation target from 2% to 4%. 2% is chosen to be the baseline level of the inflation target because that is the standard inflation target among developed countries. The impact of raising the inflation target by 2p.p. to 4% is investigated since that is the most commonly proposed adjustment ([Blanchard et al. \(2010\)](#), [Ball \(2014\)](#), [Krugman \(2014\)](#)).

Figure 5 shows the impact of the policy experiment on the supply and demand for relative assets. The impact of raising the inflation target has exactly the same qualitative impact as in section 2. The supply of assets shifts left since a lower markup lowers profits and thus the value of firms. Therefore, there are fewer assets available for households to hold. It does not shift the demand for assets by households. Observe that a shift left in the supply of savings lowers the equilibrium real rate and equilibrium relative assets. The intuition for the fall in the equilibrium real rate is also the same as in the simplified model. Households rely upon savings to consume when they are old. A fall in savings means that households consume relatively less when they are old so the price of saving rises which is equivalent to a fall in the equilibrium real rate.

Figure 6 shows the impact of the rise in inflation on the consumption path of agents across their lives. A rise in the inflation target lowers the consumption of the old relative to the young. This is because agents save less for when they are old as a result of the lower supply of assets.

Table 1 shows the numerical impact of the policy experiment with the default calibration.<sup>26</sup>

<sup>26</sup>By row, the mathematical expressions for what table 1 shows are:  $100\Delta_\pi m$ ,  $100\Delta_\pi(\log(\sum_{i=120}^{239} C_i) - \log(\sum_{i=0}^{119} C_i))$ ,  $100(\Delta_\pi(\log(K_{olg}) - \log(K_{ra})))$ ,  $100\Delta_\pi r$ . The change in capital is relative to the case when  $\bar{r}$  does not change (which includes the representative case) since I want to demonstrate that agents choose to hold more capital than when  $\bar{r}$  does not change.

Figure 5: Baseline Calibration: Asset Supply and Demand

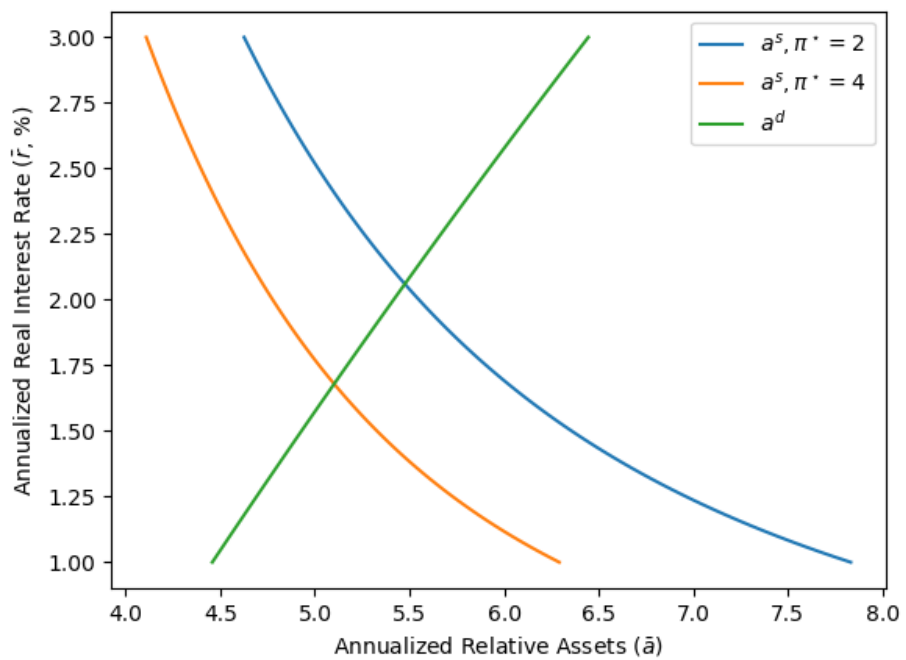


Figure 6: Baseline Calibration: Consumption Path

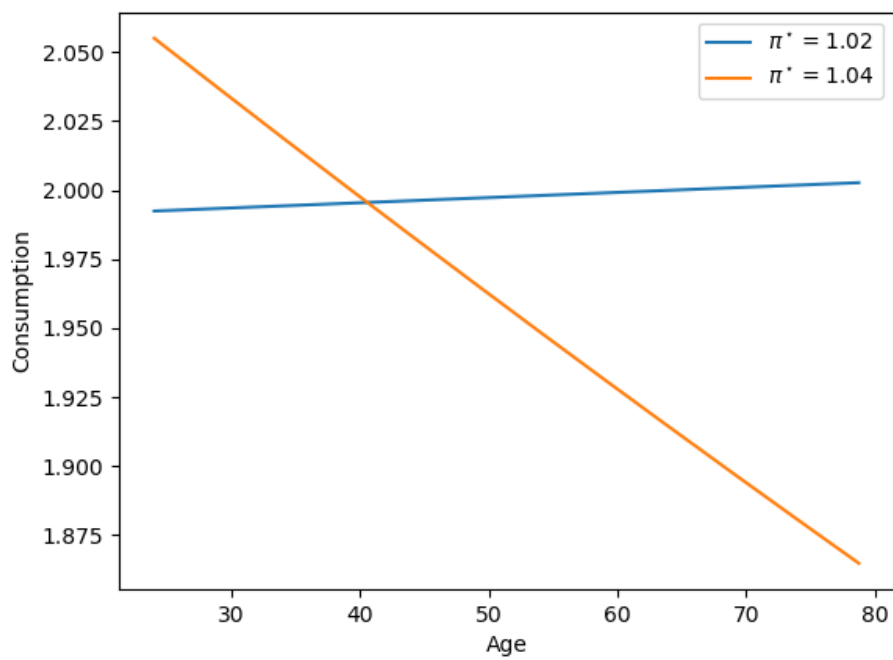




Table 1: Policy Experiment with Default Calibration

Defaults	
Change in markup (p.p.)	-1.07
Change in $\frac{C_{old}}{C_{young}}$ (%)	-5.14
Change in K (%)	4.46
Change in r (p.p.)	-0.38

A rise in the inflation target leads to a fall in the markup of 1.07p.p. This is just the first part of the channel where when inflation increases firms set a lower markup due to price rigidities. Ceteris paribus, a fall in savings reduces the ability of older agents to consume. Therefore, the consumption of the old falls relative to the young. The second row of table 1 shows that the consumption of the older half of consumers falls by 5.14 percent relative to the younger half of consumers. Next, observe that agents hold 4.46 percent more capital (relative to the representative agent baseline case). Agents want to save more in capital to try to reduce their loss of consumption when they are old. Since agents want to save more, the price of saving (the equilibrium real rate) falls. In this case, it falls by 0.38p.p.

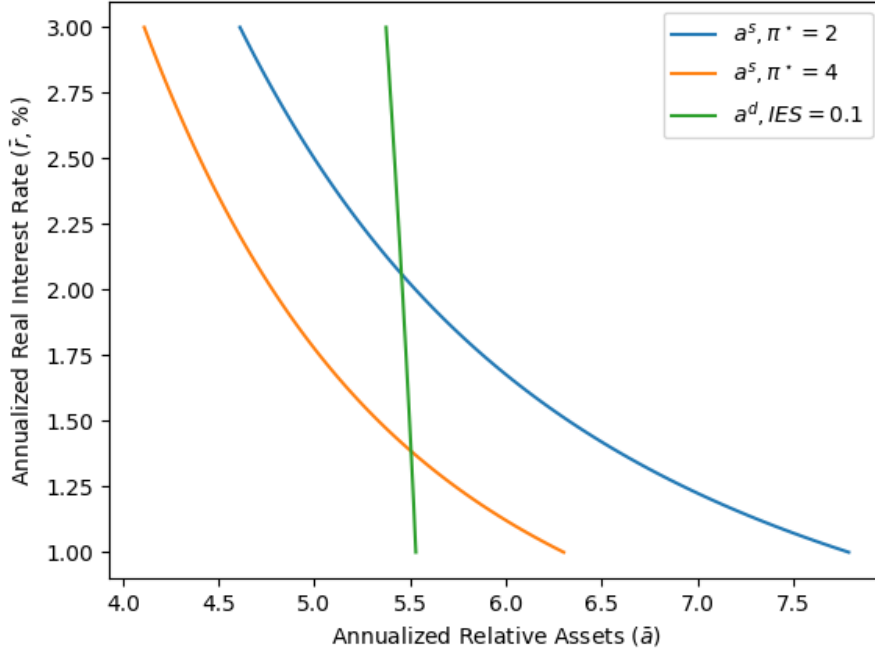
Raising the inflation target would be less effective in reducing the probability of the ZLB. A rise in the inflation target of 2p.p. only raises the equilibrium nominal interest rate by 1.62p.p. in the default calibration, as opposed to the 2p.p. rise widely assumed and predicted by standard models. Since nominal interest rates would rise by less, this would give policymakers less room to cut in bad times before hitting the ZLB.

Figure 7 shows how lowering the intertemporal elasticity of substitution (IES) to 0.1 affects the supply and demand of relative assets. Recent estimates from Best et al. (2018) suggest that the IES is 0.1. Lowering the IES to 0.1 (from its baseline value of 0.5) causes the demand for assets to tilt backwards. The reason for this is because when agents have low IES, they have a stronger desire to consume the same each period. When the real interest rate rises, agents get a higher return on their savings allowing them to consume more when they are old. With a low IES, they will then reduce the amount they save to rebalance consumption back to when they are young. In this sense, the income effect of raising the real rate dominates when IES is high enough.

Table 2 shows the numerical impact of the policy experiment under different IES.<sup>27</sup> Note that the column with an IES of 0.5 is just the default calibration and matches the results in table 1. A lower IES which pushes agents to consume the same in each period implies that, when consumption of the old falls, the price of savings rises by more and thus the return on savings falls by more. With an IES of 0.1, the equilibrium real rate falls by 0.67p.p. compared to 0.38p.p. in the baseline case. Agents can mitigate the fall in their consumption when they are old by raising their investment in capital. Since agents with low IES really want to consume the same over time, they invest more in capital hence why it rises by 8.02% relative to the representative agent case

<sup>27</sup>The rows have the same mathematical expressions as footnote 26.

Figure 7: IES = 0.1: Asset Supply and Demand



compared to 4.46% in the baseline calibration.<sup>28</sup>

Table 3, explores the impact of allowing for endogenous labor supply.<sup>29</sup> Allowing for endogenous labor leads to a somewhat smaller fall in the real interest rate than in the baseline calibration with exogenous labor (figure 5). The reason for this is that when the markup falls and thus savings falls so that agents consume less when they are old relative to when they are young, agents can choose to work more when they are old to substitute for the loss in consumption when they are old. The extent to which they do this depends upon the elasticity of labor supply. With a low

Table 2: Impact of Changing Intertemporal Elasticity of Substitution on Results of Policy Experiment

IES ( $\frac{1}{\gamma}$ )	0.1	0.2	0.3	0.5	1
Change in markup (p.p.)	-1.04	-1.05	-1.06	-1.07	-1.09
Change in $\frac{C_{old}}{C_{young}}$ (%)	-1.83	-3.11	-4.01	-5.14	-6.35
Change in K (%)	8.02	6.80	5.84	4.46	2.74
Change in r (p.p.)	-0.67	-0.58	-0.50	-0.38	-0.24

<sup>28</sup>The markup falls by less when IES is lower. This is because when IES is lower, the real interest rate falls by more which implies discounting falls and thus there's the impact of inflation on the markup is lessened slightly.

<sup>29</sup>Change in  $\frac{L_{old}}{L_{young}}$  has the following mathematical expression:  $100\Delta\pi(\log(\sum_{i=120}^{239} L_i) - \log(\sum_{i=120}^{240} L_i))$ . The other rows have the same mathematical expressions as footnote 26.

Table 3: Impact of Changing Elasticity of Labor Supply on Results of Policy Experiment

Elasticity of Labor Supply ( $\eta$ )	1	2	3	5	10
Change in $\frac{C_{old}}{C_{young}}$ (%)	-2.85	-3.68	-4.07	-4.45	-4.77
Change in $\frac{L_{old}}{L_{young}}$ (%)	4.65	3.01	2.22	1.45	0.78
Change in K (%)	2.17	2.91	3.28	3.67	4.02
Change in r (p.p.)	-0.21	-0.27	-0.30	-0.33	-0.35

elasticity of labor supply, they work 4.65% relatively more when they are old than when they are young.<sup>30</sup> Consequently, with a low elasticity of labor supply, households also choose to invest relatively less in capital since they are replacing consumption when they are old through working when they are older instead. The fall in the equilibrium real rate lessens when households have a low elasticity of labor supply. As the elasticity of labor supply gets large, the model converges back to the exogenous labor supply case in table 1.

I also explore the impact of idiosyncratic labor shocks. Appendix G examines how idiosyncratic labor shocks can be embedded within the New Keynesian life cycle model of section 3. Figure 8 shows the impact of adding idiosyncratic labor shocks. The blue, orange and green curves are identical to figure 5. The blue and orange curves are the supply of relative assets respectively before and after the shift left in relative asset supply due to a rise in the inflation target. The green curve is the demand for relative assets with OLG households and no idiosyncratic labor shocks. The dashed red curve represents the OLG model with idiosyncratic labor included. The only effective impact is that the demand for relative assets shifts out. The intuition for this is that households face more risk so they want to save more as a precaution against this risk. However, there does not appear to be a substantive impact on the degree to which the real interest rate falls following the shift left in the supply of assets.

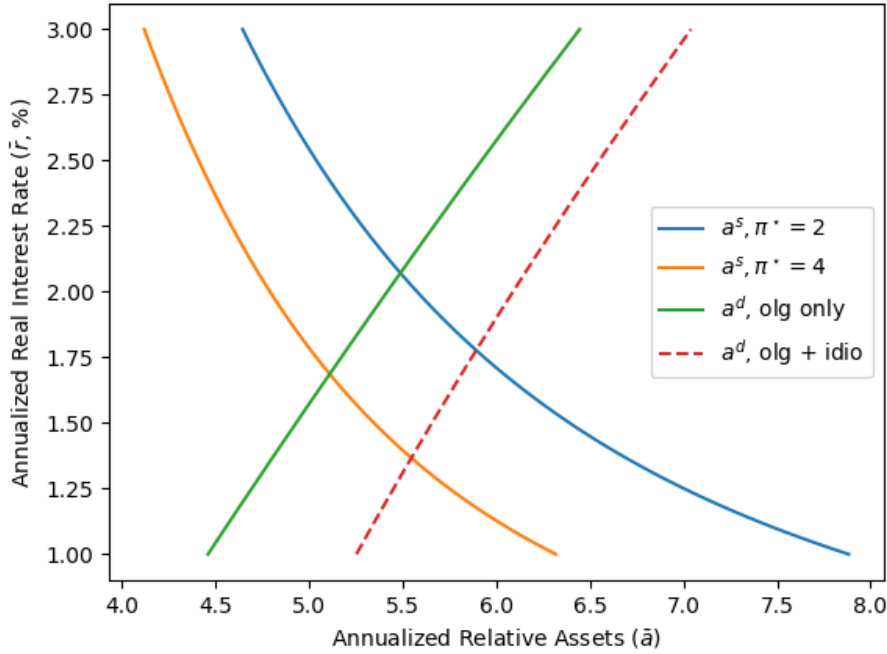
## 6 Optimal Inflation Target

In this section, I analyse the optimal inflation target. The main aim here is to investigate how the optimal inflation target changes in response to a fall in the equilibrium real interest rate. I am interested in this because much of the motivation for raising the inflation target has focused upon the apparent recent fall in the equilibrium real rate in much of the developed world which makes the probability of hitting the ZLB more likely. I also investigate the level of the optimal inflation target.

To consider these questions, the economy is simulated over 1,000 periods (250 years). A set of shocks is drawn from the calibrated shock distributions and these same shocks are used in every simulation. The welfare (the population-weighted utility averaged across all periods) is then computed under different inflation targets. To capture the impact of a fall in the equilibrium

<sup>30</sup>Define old agents to be those above the average age and those who are young to be below the average age.

Figure 8: OLG with Idiosyncratic Labor Shocks: Asset Supply and Demand



real rate, we investigate what happens when agents' premium on safe bonds rises by  $2p.p.$  which implies that  $\xi = -0.02$ . I choose to model a fall in the equilibrium real rate as an increase in the desire for safe assets because this is often used to explain the fall in the equilibrium real rate and because this does not have any impact upon asset demand or asset supply so it is easier to isolate the impact of the change.

The welfare under the baseline calibration is displayed in figure 9. The blue line represents the welfare under the baseline equilibrium real rate (2.07%). The orange line represents the welfare under a  $2p.p.$  lower equilibrium real rate. For a low inflation target, the welfare under the lower equilibrium real rate is lower. This is because under low levels of inflation a lower equilibrium real rate implies a higher probability of hitting the ZLB which implies lower welfare. However, for a higher inflation target, the welfare is identical. Figure 10 shows the probability of hitting the ZLB in each case. The probability is higher for the lower equilibrium real rate case when the inflation target is low. However, once the inflation target is high enough, there is no probability of hitting the ZLB in either case so the welfare is identical. The fall in the equilibrium real rate implies that the optimal inflation target rises by  $0.3p.p.$  from 0.6% to 0.9%.

The baseline calibration does not generate shocks that hit the ZLB very frequently. Larger shocks are allowed for by scaling up each of the shock standard deviations by a factor of 1.5. The revised welfare and probability of hitting the ZLB are shown in figure 11 and figure 12 respectively. The larger shocks mean that under a low equilibrium real rate, there is a higher probability of hitting the ZLB. This implies that the optimal inflation target rises by more when the equilibrium

Figure 9: Welfare under Baseline Calibration

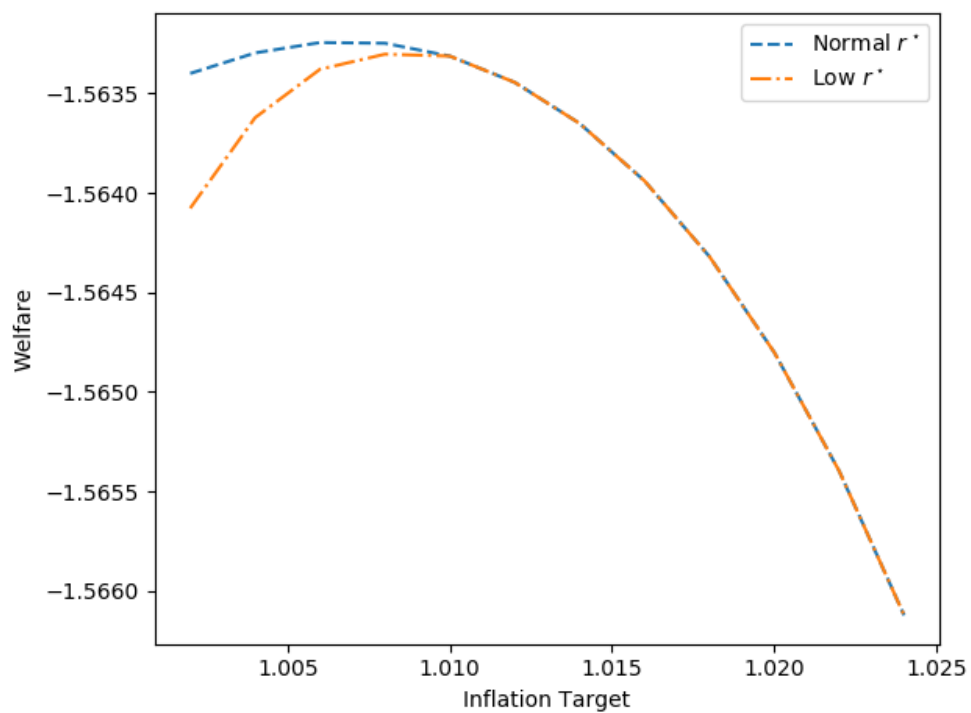


Figure 10: Probability of Hitting the ZLB under Baseline Calibration

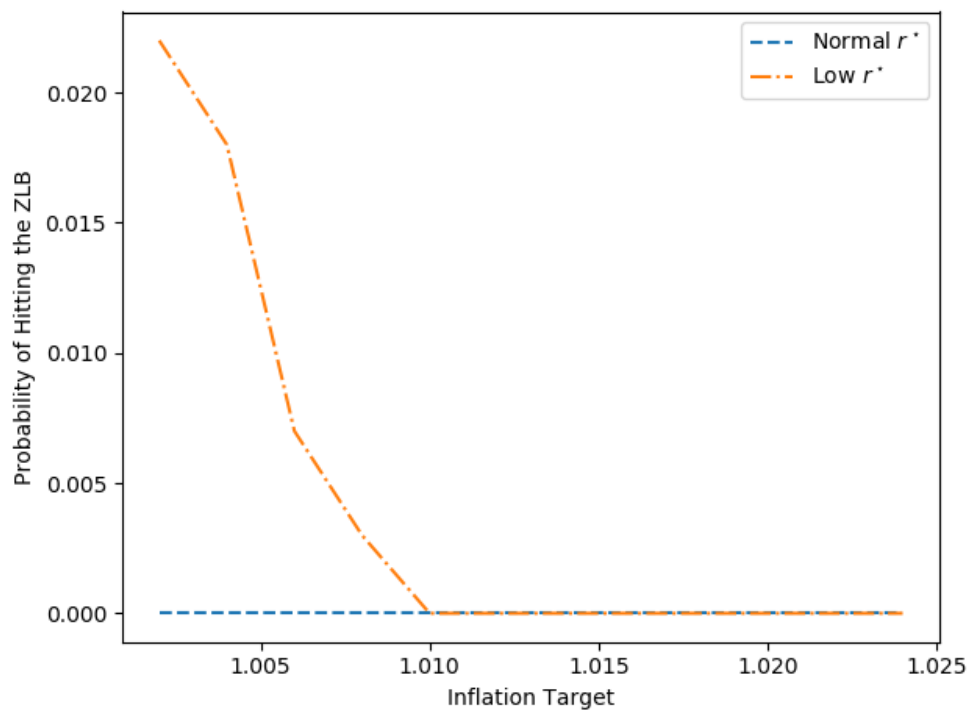
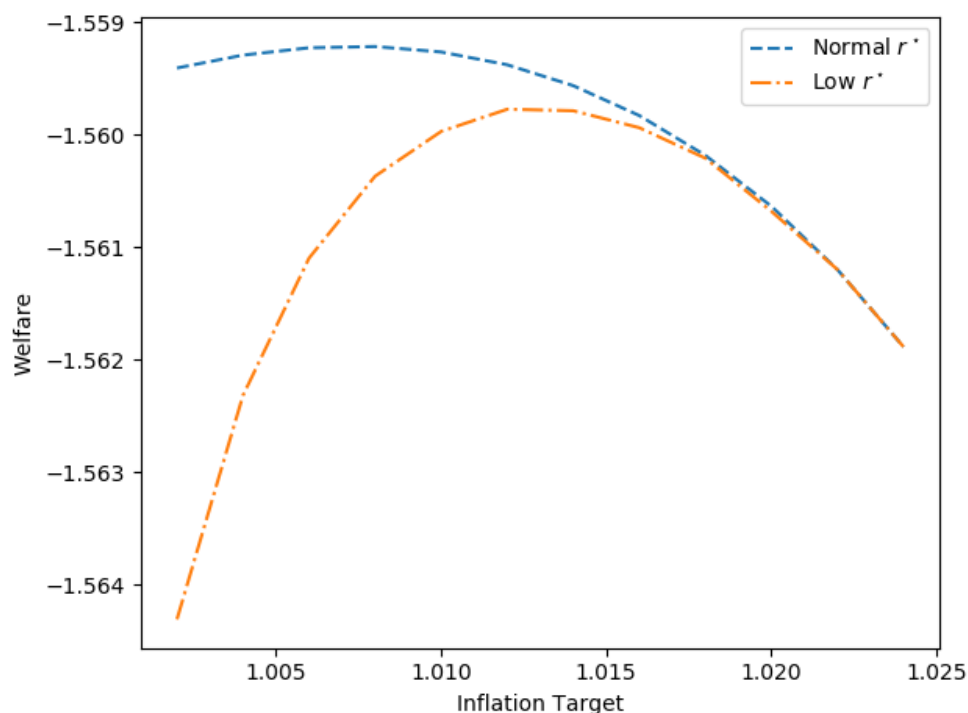


Figure 11: Welfare under Larger Shocks



real rate falls. The fall in the equilibrium real rate of 2p.p. leads to a rise of 0.6p.p. in the optimal inflation target from 0.6p.p. to 1.2p.p.

The simulations imply that the fall in the equilibrium real rate does not generate a large increase in the optimal inflation target. In the high shock case, the increase was only 0.6p.p. The intuition for why is that the probability of hitting the zero lower bound falls quickly when the inflation target is raised. Thus, the benefits of raising the inflation target are quickly outweighed by the increased costs of price dispersion.

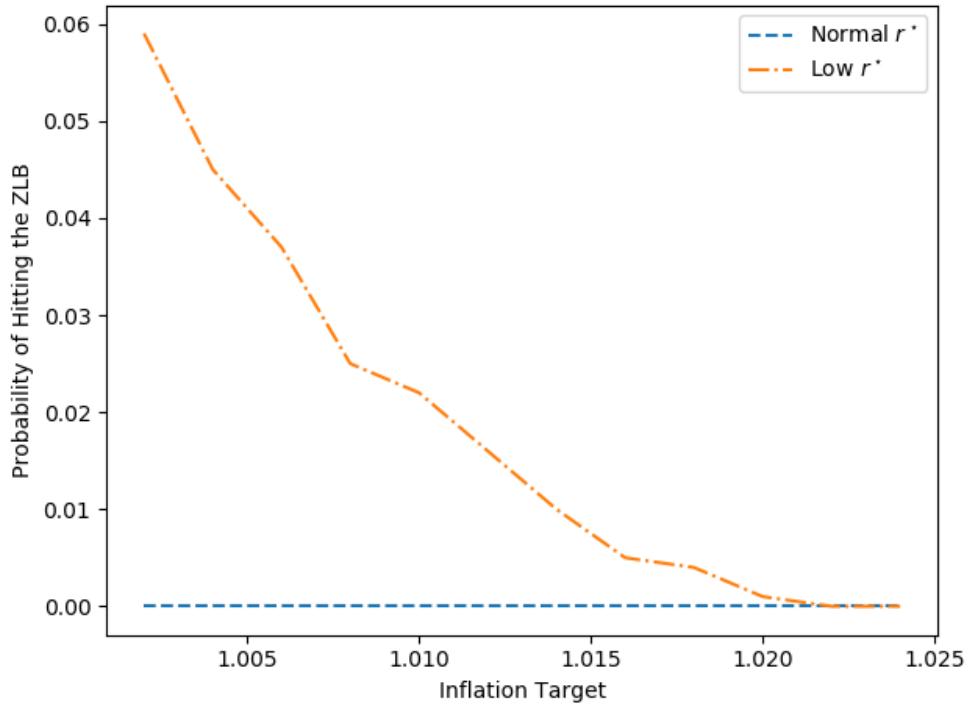
In all cases that have been considered, the inflation target is not much more than 1%. This is similar to the representative agent case. The optimal inflation target is low because of the high costs of price dispersion. Even once the size of the shocks is increased and the ZLB binds more, the costs of inflation through higher price dispersion seem to dominate the benefits of avoiding the ZLB. This also appears to be true in the representative case since [Coibion et al. \(2012\)](#) compute an optimal inflation target of around 1% in a representative agent model with the ZLB.

## 7 Empirical Evidence

In this section, I provide reduced form empirical estimates of two key relationships in my paper to complement existing structural analysis.

I first consider the relationship between long-run inflation and the equilibrium real rate. The

Figure 12: Probability of Hitting the ZLB under Larger Shocks



mechanism I propose implies that a rise in long-run inflation lowers the equilibrium real rate. There is existing empirical evidence for this. Both [King and Watson \(1997\)](#) and [Rapach \(2003\)](#) find such a relationship using structural VAR methods. To complement this existing evidence, I conduct a reduced form analysis of the relationship.

A problem with studying the reduced form relationship between long-run inflation and the equilibrium real rate is correlated trends. Inflation has trended down in recent years at the same time as the equilibrium real rate has fallen. If the fall in inflation was the only reason for the fall in the equilibrium real rate then this would imply my channel is incorrect. However, there are many other reasons why the equilibrium real rate has fallen. Therefore, since real rates have fallen at the same time as inflation has fallen but for reasons other than the fall in inflation, a simple regression of the equilibrium real rate on inflation is likely to produce a positively biased coefficient.

To overcome common trends in inflation and real rates, I conduct panel data regressions with time fixed effects. Using time fixed effects means that the common global trend in real rates can be controlled for. It is then possible to assess whether higher relative inflation is associated with a positive or negative deviation from the global trend in real rates. If other factors that cause deviations from the global trend in real rates for a country are uncorrelated with that country's inflation level then this relationship is causal.

Regression: Equation 73 shows the estimated model.  $\alpha_i$  represents country fixed effects i.e. whether the real interest rate is systematically higher in a country.  $\delta_t$  represents the time fixed effects.  $\beta$  is the coefficient of interest which represents the change in the real interest rate relative



Table 4: Empirical Estimates of Relationship between Long-Run Inflation and the Equilibrium Real Rate

$RealRate_{i,t}^{10yr}$	(1)	(2)	(3)	(4)
$Inflation_{i,t-4,t}$	-0.167*** (0.040)	-0.196*** (0.044)	-0.607*** (0.069)	-0.904*** (0.149)
country dummies		*	*	*
year dummies			*	*
econ controls				*
N	1151	1151	1151	833

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.

to the global trend associated with a 1p.p. rise in long-run inflation. Controls are also included.

$$r_{i,t} = \alpha_i + \delta_t + \beta \overline{Inflation}_{i,t} + \Gamma Controls_t + u_{i,t} \quad (73)$$

The panel is limited to OECD members. Annual data is used. Long-run inflation ( $\overline{Inflation}_{i,t}$ ) is measured as the moving average of the current and previous four years of CPI inflation.<sup>31</sup> <sup>32</sup> The real interest rate is measured by the 10 year expected return on safe bonds. The 10 year real rate is used since there is more data availability and its likely to be a much less noisy measure of the equilibrium real rate. To measure the 10 year real rate the measure of long-run inflation is subtracted from the nominal interest rate on 10 year government bonds.<sup>33</sup> I also allow for business cycle controls.<sup>34</sup> The business cycle controls are set to be GDP growth and change in unemployment at  $t$  and  $t - 1$ .

The results are given in table 4. Without fixed or time effects a 1p.p. rise in long-run inflation is associated with a fall of  $-0.17$ p.p. in the real rate. This falls slightly once country fixed effects are introduced. It has already been noted that inflation and the real rate both have a negative trend so it is unsurprising that once time fixed effects which remove this source of positive association are added, the coefficient drops a lot to  $-0.61$ p.p. Controls increase the strength of the impact. A causal interpretation of the regression without controls is that a rise of 1p.p. in long-run inflation lowers the equilibrium real rate by 0.61p.p.

The second relationship I am interested in is the relationship between long-run inflation and the long-run markup. The first part of the new channel I propose implies that a rise in long-run

<sup>31</sup>It may seem strange that the regressor is not the inflation target but nearly all inflation targets have not changed since they were introduced so the inflation target would be almost completely captured by the country fixed effects ( $\alpha_i$ ).

<sup>32</sup>Varying the measure to a different moving average does not appear to impact the results.

<sup>33</sup>Computing the measure of 10 year real interest rates by subtracting current inflation (rather than the measure of long-run inflation) from the nominal interest rate on 10 year government bonds yields similar results.

<sup>34</sup>It is undesirable to have controls that capture the long-run state of the economy since these could interfere in the long-run relationship between inflation and the real rate. Business cycle controls are short-term and should not generate this problem.

inflation lowers the long-run markup. There is existing empirical evidence for this. Both [Bénabou \(1992\)](#) and [Banerjee and Russell \(2001\)](#) find such a relationship using structural vector error correction models. To complement this existing evidence a reduced form analysis is conducted.

The labor share is used as a proxy for the markup. The markup cannot be directly measured. However, equation 74 shows that that a rise in the labor share of 1p.p. is equivalent to a  $\frac{1}{1-\alpha}$ p.p. rise in the inverse of the markup. And since  $m > 1$  a  $\frac{1}{1-\alpha}$ p.p. rise in the inverse of the markup equates to a greater than  $\frac{1}{1-\alpha}$ p.p. fall in the markup.<sup>35</sup> Therefore, if a 1p.p. in inflation raises the labor share by  $x$ p.p. then it should lower the markup by at least  $\frac{1}{1-\alpha}x$ p.p.

$$\frac{1}{\bar{m}} = \frac{1}{1-\alpha} \frac{\bar{W}\bar{L}}{\bar{Y}} \quad (74)$$

Equation 75 shows the regression relationship. It uses the same basic panel data structure as for the inflation-real rate relationship.

$$LaborShare_{i,t} = \alpha_i + \delta_t + \beta \overline{Inflation}_{i,t} + \Gamma Controls_t + u_{i,t} \quad (75)$$

The labor share is measured as the percentage net value added in production that is received as compensation by employees. I compute this for firms only since we are interested in measuring the markup which would be most related to firms' labor share. This data is taken from National Accounts at the UN and the OECD.<sup>36</sup>

The results are given in table 5. They do not vary very much once country fixed effects are introduction. A 1p.p. rise is associated with a 0.3p.p. rise in the markup. Assuming causality and  $\alpha = 0.35$ , observe that a 1p.p. rise in the markup leads to a fall of more than 0.46p.p. in the markup. Once controls are added, the results are not significant due to a large increase in the standard error but I do not think this is concerning since the coefficient itself is unchanged and we are primarily interested in the third column regression without controls.

I verify these two relationships are robust. Tables 7 and 8 verifies the relationships continue to hold with just OECD members that joined before 1975 (these regressions exclude a number of mostly Eastern European countries that joined from the 1990s onwards). Tables 9 and 10 verifies the relationships still apply under low inflation. Tables 11 and 12, looks at whether the result remain for the period before 2000 only. Tables 13 and 14 analyzes whether the relationship continues to hold during/after 2000 only. Spurious regressions are generally considered to be less of a problem in panel data since we can control for common trends across countries by time fixed effects and idiosyncratic trends within countries are unlikely to drive results. However, I verify in tables 15 and 16 that the results still hold after differencing.

<sup>35</sup>For example, when  $m_0 = 2$ ,  $m_1 = 1.98$ , observe that  $\frac{1}{m_0} = 0.5$ ,  $\frac{1}{m_1} \approx 0.51$ . In this case, a 1p.p. rise in the inverse of the markup is equivalent to a 2p.p. fall in the markup.

<sup>36</sup>Both the OECD and the UN National Accounts data for firms has some gaps. I take the UN data by default and fill in gaps with OECD data.

Table 5: Empirical Estimates of Relationship between Long-Run Inflation and long-run Markup

$LaborShare_{i,t}$	(1)	(2)	(3)	(4)
$Inflation_{i,t-4,t}$	0.209*	0.304***	0.281***	0.298
	(0.083)	(0.037)	(0.032)	(0.178)
country dummies		*	*	*
year dummies			*	*
econ controls				*
N	813	813	813	721

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.

## 8 Conclusion

Many economists have proposed raising the inflation target in recent years. They argue that this will raise the average nominal interest rate and thus reduce the probability of hitting the ZLB. It is generally assumed and a feature of standard models that raising the inflation target has no impact upon the equilibrium real rate.

In this paper, I show that once heterogeneity is introduced into a standard New Keynesian model through either generational features or idiosyncratic shocks, raising the inflation target will lower the equilibrium real rate. In my baseline calibration of a New Keynesian model with life cycle features, a rise in the inflation target from 2% to 4% lowers the equilibrium real rate by 0.38*p.p.*. Many of the arguments for raising the inflation target are premised upon the perceived fall in the equilibrium real rate in recent years. I find that a fall of 2*p.p.* in the equilibrium real rate within my framework, raises the optimal inflation target by 0.3 – 0.6*p.p.*

The channel I propose is empirically realistic. It relies upon the existence of price stickiness (or other forms of price rigidity) and the existence of a consumer life cycle. Both of these features are observable in the real-world. There is already structural econometric evidence for my channel. I provide additional reduced form evidence which further supports its existence.

The results of this paper provide valuable insights for policy-makers. A rise in the inflation target will lower the equilibrium real rate and therefore lead to a smaller increase in average nominal interest rates than is generally believed. This implies that raising the inflation target is likely to be less effective in reducing the probability of hitting the ZLB than expected. And my welfare simulations imply that a fall in the equilibrium real rate is unlikely to justify a large increase in the inflation target.

Going forward, my results imply that central banks should look at alternatives to raising the inflation target. Frameworks exist that could help to boost inflation expectations during recessions without raising long-run inflation. Price level targeting, either just for a short-term period post-recession or for all periods, appears to work theoretically. Another alternative is nominal GDP targeting, although this would have to be updated periodically to account for

changes in the trend of real GDP growth.. However, although these targeting frameworks could raise expectations of inflation during a recession, they do not solve the problem that central banks will likely enter future recessions with low nominal interest rates. How central banks can best tackle a slump with their main policy tool dramatically constrained remains a question that deserves more investigation.

# Appendices

## A Intuition in a Simplified Model Details

In this section, additional details are provided relating to section 2.

### A.1 Calvo Pricing: Inflation and the Markup

Here, the standard Calvo pricing setup is assumed and the relationship between inflation and the markup is derived. This is an example of the inflation-markup relationship that is discussed in section 2.1

There are a continuum of intermediate firms indexed by  $i$  which produce differentiated intermediate goods. There is a competitive final goods firm with constant elasticity of substitution (CES) production with CES parameter  $\sigma$ . The final goods firm's demand for good  $i$  is represented by:

$$Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\sigma} Y_t \quad (76)$$

where:

$$\int_0^1 P_{i,t} Y_{i,t} di = P_t Y_t \quad (77)$$

$$P_t = \left( \int_0^1 P_{i,t}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \quad (78)$$

$$Y_t = \left( \int_0^1 Y_{i,t}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

Intermediate firms have some probability  $\lambda$  of updating their price each period. Firms have the following maximisation problem:

$$\max_{P_t^*, \{Y_{t+k}^*\}_{k=0}^{\infty}} \mathbb{E}_t \left[ \sum_{k=0}^{\infty} (1-\lambda)^k \beta^k \left[ \frac{P_t^* Y_{t+k}^*}{P_{t+k}} - MC_{t+k} P_{t+k} Y_{t+k}^* \right] \right] \quad (79)$$

s.t.  $\forall k$ :

$$Y_{t+k}^* = Y_{t+k} \left( \frac{P_t^*}{P_{t+k}} \right)^{-\sigma} \quad (80)$$

Inputting equation 80 into equation 79 and taking FOCs yields:

$$\mathbb{E}_t \left[ \sum_{k=0}^{\infty} (1-\lambda)^k \beta^k \left( (1-\sigma) P_t^{*\sigma-1} Y_{t+k} P_{t+k}^{\sigma-1} + \sigma MC_{t+k} Y_{t+k} P_t^{*\sigma-1} P_{t+k}^{\sigma} \right) \right]$$

In steady state, this can be rewritten as:

$$\sum_{k=0}^{\infty} (1-\lambda)^k \beta^k \bar{\Pi}^{k\sigma} \left[ \left( \frac{P^*}{P} \right) \frac{1}{\bar{\Pi}^k} - \frac{\sigma}{\sigma-1} \overline{MC} \right]$$

This can be further simplified to get:

$$\overline{MC} = \frac{\sigma-1}{\sigma} \frac{1 - (1-\lambda)\beta\bar{\Pi}^\sigma}{1 - (1-\lambda)\beta\bar{\Pi}^{\sigma-1}} \overline{\left( \frac{P^*}{P} \right)} \quad (81)$$

A price dispersion term is defined:

$$\nu_t = \int_0^1 \left( \frac{P_{i,t}}{P_t} \right)^{-\sigma} di \quad (82)$$

This can be rewritten as:

$$\nu_t = \lambda \left( \frac{P_t^*}{P_t} \right)^{-\sigma} + (1-\lambda)\nu_{t-1}\Pi_t^\sigma$$

In steady state:

$$\bar{\nu} = \frac{\lambda}{1 - (1-\lambda)\bar{\Pi}^\sigma} \overline{\left( \frac{P^*}{P} \right)}^{-\sigma}$$

Multiplying both sides of equation 81 by  $\bar{\nu}$  yields:

$$\overline{MC}\bar{\nu} = \frac{\sigma-1}{\sigma} \frac{1 - (1-\lambda)\beta\bar{\Pi}^{\sigma-1}}{1 - (1-\lambda)\beta\bar{\Pi}^{\sigma-1}} \overline{\left( \frac{P^*}{P} \right)}^{1-\sigma} \frac{\lambda}{1 - (1-\lambda)\bar{\Pi}^\sigma} \quad (83)$$

Next, we note that equation 78 can be rewritten as

$$1 = \lambda \left( \frac{P_t^*}{P_t} \right)^{1-\sigma} + (1-\lambda)\Pi_t^{\sigma-1}$$

where  $\Pi_t = \frac{P_t}{P_{t-1}}$  and also  $\pi_t = \Pi_t - 1$ . In steady state:

$$\left( \frac{P^*}{P} \right)^{1-\sigma} = \frac{1}{\lambda} [1 - (1-\lambda)\bar{\Pi}^{\sigma-1}] \quad (84)$$

Inputting equation 84 into equation 83 and simplifying yields:

$$\overline{MC}\bar{\nu} = \frac{\sigma-1}{\sigma} \left[ \frac{1 - (1-\lambda)\bar{\Pi}^{\sigma-1}}{1 - (1-\lambda)\beta\bar{\Pi}^{\sigma-1}} \right] \left[ \frac{1 - (1-\lambda)\beta\bar{\Pi}^\sigma}{1 - (1-\lambda)\bar{\Pi}^\sigma} \right] \quad (85)$$

We set  $\bar{m} = (\overline{MC}\bar{\nu})^{-1}$  which is a measure of the effective markup (this is explained in ap-

pendix A.2). Therefore, we can rewrite equation 85 as:

$$\bar{m} = \frac{\sigma}{\sigma - 1} \left[ \frac{\bar{\Pi}^{1-\sigma} - (1-\lambda)\beta}{\bar{\Pi}^{1-\sigma} - (1-\lambda)} \right] \left[ \frac{\bar{\Pi}^{-\sigma} - (1-\lambda)}{\bar{\Pi}^{-\sigma} - (1-\lambda)\beta} \right] \quad (86)$$

Raising  $\bar{\Pi}$  increases the first square bracket but lowers the second. When  $\beta < 1$ , the second square bracket dominates since  $\bar{\Pi}^{-\sigma}$  changes by more than  $\bar{\Pi}^{1-\sigma}$  so raising average inflation lowers the markup.

By average markup, I'm effectively referring to the average markup weighted by sales. This is why I get qualitatively different results to [Ascari and Sbordone \(2014\)](#) who consider the non-weighted average markup and show that it can rise when inflation rises.

**Formal: Raising Inflation Lowers the Markup** The result that raising inflation lowers the markup when  $\beta < 1$  is now shown formally. First derivatives are applied:

$$\frac{d\bar{m}}{d\bar{\Pi}} = \left[ \frac{(1-\sigma)\bar{\Pi}^{-\sigma}}{\bar{\Pi}^{1-\sigma} - (1-\lambda)\beta} - \frac{(1-\sigma)\bar{\Pi}^{-\sigma}}{\bar{\Pi}^{1-\sigma} - (1-\lambda)} - \frac{\sigma\bar{\Pi}^{-\sigma-1}}{\bar{\Pi}^{-\sigma} - (1-\lambda)} + \frac{\sigma\bar{\Pi}^{-\sigma-1}}{\bar{\Pi}^{-\sigma} - (1-\lambda)\beta} \right] \frac{\sigma}{\sigma - 1} \left[ \frac{\bar{\Pi}^{1-\sigma} - (1-\lambda)\beta}{\bar{\Pi}^{1-\sigma} - (1-\lambda)} \right] \left[ \frac{\bar{\Pi}^{-\sigma} - (1-\lambda)}{\bar{\Pi}^{-\sigma} - (1-\lambda)\beta} \right]$$

Concentrating on the first square bracket:

$$\frac{d\bar{m}}{d\bar{\Pi}} \propto - \frac{(1-\sigma)\bar{\Pi}^{-\sigma}(1-\lambda)(1-\beta)}{(\bar{\Pi}^{1-\sigma} - (1-\lambda)\beta)(\bar{\Pi}^{1-\sigma} - (1-\lambda))} - \frac{\sigma\bar{\Pi}^{-\sigma-1}(1-\lambda)(1-\beta)}{(\bar{\Pi}^{-\sigma} - (1-\lambda))(\bar{\Pi}^{-\sigma} - (1-\lambda)\beta)}$$

Rearranging and remove  $(1-\lambda)(1-\beta)$  (which would not be possible if  $\beta = 1$ ):

$$\frac{d\bar{m}}{d\bar{\Pi}} \propto \frac{\bar{\Pi}^{-\sigma}}{(\bar{\Pi}^{1-\sigma} - (1-\lambda)\beta)(\bar{\Pi}^{1-\sigma} - (1-\lambda))} - \frac{\sigma}{\sigma - 1} \frac{\bar{\Pi}^{-\sigma-1}}{(\bar{\Pi}^{-\sigma} - (1-\lambda))(\bar{\Pi}^{-\sigma} - (1-\lambda)\beta)}$$

Substitute out  $\frac{\sigma}{\sigma-1}$  using equation 86 and rearrange to get:

$$\frac{d\bar{m}}{d\bar{\Pi}} \propto \bar{\Pi}^{-\sigma}(\bar{\Pi}^{-\sigma} - (1-\lambda))^2 - \bar{m}\bar{\Pi}^{-\sigma-1}(\bar{\Pi}^{1-\sigma} - (1-\lambda))^2$$

Simplifying:

$$\frac{d\bar{m}}{d\bar{\Pi}} \propto \bar{\Pi}^{-\sigma}(\bar{\Pi}^{-\sigma} - (1-\lambda))^2 - \bar{m}\bar{\Pi}^{-\sigma-1}(\bar{\Pi}^{1-\sigma} - (1-\lambda))^2$$

$\bar{m} \geq 1$  otherwise firms exit the market and  $\frac{d\bar{m}}{d\bar{\Pi}}$  is decreasing in  $\bar{m}$ . Therefore,  $\frac{d\bar{m}}{d\bar{\Pi}}$  takes its highest possible value when  $\bar{m} = 1$ . I show that even in this case  $\frac{d\bar{m}}{d\bar{\Pi}} < 0$  and thus the markup always decreases in inflation. Under  $\bar{m} = 1$ , simplify to yield:

$$\frac{d\bar{m}}{d\bar{\Pi}}_{\bar{m}=1} \propto \bar{\Pi}^{-3\sigma} + (1-\lambda)^2\bar{\Pi}^{-\sigma} - [\bar{\Pi}^{1-3\sigma} + \bar{\Pi}^{-\sigma-1}(1-\lambda)^2]$$

$$\frac{d\bar{m}}{d\bar{\Pi}} \Big|_{\bar{m}=1} \propto (\bar{\Pi} - 1)((1 - \lambda)^2 - \bar{\Pi}^{-2\sigma})$$

Then note that by equation 86  $\bar{\Pi}^{-\sigma} \geq 1 - \lambda$  since this is needed to guarantee  $\bar{m}$  is positive. Thus,  $\frac{d\bar{m}}{d\bar{\Pi}} < 0$  so I have demonstrated that when average inflation rises, the average markup always falls under Calvo pricing when  $\beta < 1$ .

## A.2 Asset Demand with Price Dispersion

This section provides the derivation for the profit of the firm section 2.2 when prices are dispersed.

Intermediate firms produce output  $Y_{i,t}$  by a linear production function over labor  $L_{i,t}$ :

$$Y_{i,t} = L_{i,t} \quad (87)$$

Inputting equation 76 into equation 87 and aggregating yields:

$$Y_t \nu_t = L_t \quad (88)$$

where  $\nu_t$  is defined by equation 82 and:

$$L_t = \int_0^1 L_{i,t}$$

The real marginal cost  $MC_t$  of intermediate firms is just their wage bill ( $W_t$ ):

$$MC_t = W_t \quad (89)$$

Each intermediate firm has the following real profits  $\Omega_{i,t}$ :

$$\Omega_{i,t} = \frac{P_{i,t} Y_{i,t}}{P_t} - MC_t Y_{i,t} \quad (90)$$

Inputting equation 76 into equation 90 and aggregating (where we also apply equation 77) yields:

$$\Omega_t = (1 - MC_t \nu_t) Y_t \quad (91)$$

where:

$$\Omega_t = \int_0^1 \Omega_{i,t} di$$

Next, we define  $m_t$  to be  $\frac{1}{MC_t \nu_t}$ . We input this into equation 91:

$$\Omega_t = \left(1 - \frac{1}{m_t}\right) Y_t \quad (92)$$

We observe that  $m_t$  represents the inverse of real costs of firms, which is a measure of the average markup across firms. When  $m_t = 1$  (the competitive case), firms make no profits. However,



as the markup rises  $m_t \uparrow$ , firms make higher profits.

Next, we multiply the RHS of equation 92 by  $\frac{W_t}{MC_t}$  (which equals 1 by equation 89 and apply equation 88 before simplifying:

$$\Omega_t = \left(1 - \frac{1}{m_t} \frac{W_t L_t}{MC_t \nu_t}\right)$$

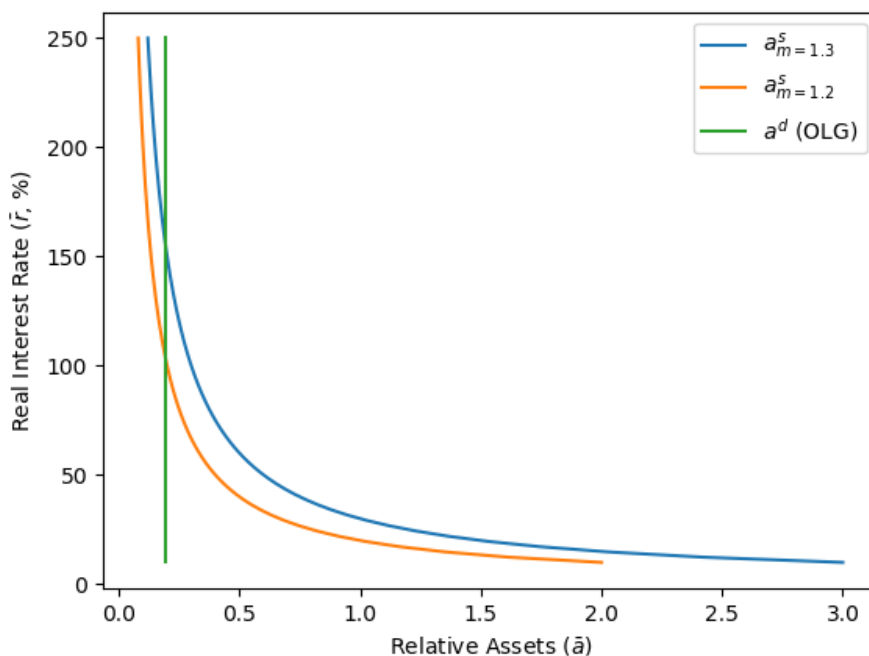
$$\Omega_t = (m_t - 1) \frac{W_t}{P_t} L_t$$

We see that we get the same as the case without price dispersion.

### A.3 OLG in a Non-Annualized Model

Figure 13 shows the case where the OLG model is non-annualized, unlike figure 3 where it is annualized. The real interest rate is very high since it represents the return from one generation to the next.

Figure 13: Equilibrium under a Fall in the Markup: 2. OLG (Not Annualized)



## B Model Details

In this section, additional details are provided on the derivations of section 3.

## B.1 Firms

### B.1.1 Cost Minimisation Details

Intermediate firms minimise their costs. They face the following problem:

$$\min_{K_{i,j,t}, L_{i,j,t}} (1 + \tau)((r_t + \tau)K_{i,j,t} + W_t L_{i,j,t})$$

s.t.

$$Y_{i,j,t} = A_t K_{i,j,t}^\alpha L_{i,j,t}^{1-\alpha}$$

Note that  $r_t, W_t$  are real variables. Setting up a Lagrangean and solving yields:

$$(1 + \tau)(r_t + \delta) = \lambda_{i,j,t} \alpha A_t K_{i,j,t}^{\alpha-1} L_{i,j,t}^{1-\alpha} \quad (93)$$

$$(1 + \tau)W_t = \lambda_{i,j,t} (1 - \alpha) A_t K_{i,j,t}^\alpha L_{i,j,t}^{-\alpha} \quad (94)$$

We can combine equations 93 and 94 to get:

$$\frac{K_{i,j,t}}{L_{i,j,t}} = \frac{\alpha}{1 - \alpha} \frac{W_t}{r_t + \delta} \quad (95)$$

We observe in equation 95 that the ratio of each firm's capital to labor ( $\frac{K_{i,j,t}}{L_{i,j,t}}$ ) is determined by aggregate variables and is therefore the same across firms. Thus:  $\frac{K_{i,j,t}}{L_{i,j,t}} = \frac{K_t}{L_t}$ .  $\lambda_{i,j,t}$  is the marginal cost after tax of production of the firm. This is constant for all firms and is equal to the real marginal cost after tax of the firm.  $MC_t$  is defined to be the marginal cost of firms before tax so that  $\lambda_{i,j,t} = (1 + \tau)MC_t$ . This yields:

$$MC_t = \frac{r_t + \delta}{\alpha A_t K_t^{\alpha-1} L_t^{1-\alpha}} \quad (96)$$

$$MC_t = \frac{W_t}{(1 - \alpha) A_t K_t^\alpha L_t^{-\alpha}} \quad (97)$$

### B.1.2 Aggregation of Cost Minimisation Conditions Details

Rewrite output as follows:

$$Y_{i,j,t} = A_t K_{i,j,t}^\alpha L_{i,j,t}^{1-\alpha}$$

$$Y_{j,t} \left( \frac{P_{i,j,t}}{P_{j,t}} \right)^{-\sigma} = A_t \left( \frac{K_{i,j,t}}{L_{i,j,t}} \right)^\alpha L_{i,j,t}$$

Taking integrals and noting that the ratio  $\frac{K_{i,j,t}}{L_{i,j,t}}$  is the same across  $i, j$ :

$$Y_{j,t} \nu_{j,t} = A_t \left( \frac{K_t}{L_t} \right)^\alpha L_{j,t}$$

$$a_j^{\frac{1}{\sigma_2}} Y_t \left( \frac{P_{j,t}}{P_t} \right)^{-\sigma_2} \nu_{j,t} = a_j A_t \left( \frac{K_t}{L_t} \right)^\alpha L_{j,t}$$

Taking integrals again to get a condition with aggregate output:

$$Y_t \int_0^1 a_j \left( \frac{P_{j,t}}{P_t} \right)^{-\sigma_2} \nu_{j,t} dj = A_t \left( \frac{K_t}{L_t} \right)^\alpha L_t$$

Applying the definition of marginal costs and inputting the lump sum transfer, equation 49 can be rewritten more simply as equation 98. Also note that the lump sum transfer equals  $\tau(r_t K_{i,j,t} + W_t L_{i,j,t})$ .

$$\begin{aligned} \Omega_{i,j,t} &= \frac{P_{i,j,t} Y_{i,j,t}}{P_t} - (r_t + \delta) K_{i,j,t} + W_t L_{i,j,t} \\ \Omega_{i,j,t} &= \frac{P_{i,j,t} Y_{i,j,t}}{P_t} - MC_t Y_{i,j,t} \end{aligned} \quad (98)$$

The same steps can then be followed with real profits:

$$\begin{aligned} \Omega_{i,j,t} &= \frac{P_{i,j,t}}{P_t} Y_{i,j,t} - MC_t Y_{i,j,t} \\ \Omega_{i,j,t} &= \frac{P_{i,j,t}}{P_{j,t}} \frac{P_{j,t}}{P_t} Y_{i,j,t} - MC_t Y_{j,t} \left( \frac{P_{i,j,t}}{P_{j,t}} \right)^{-\sigma} \\ \Omega_{j,t} &= \frac{P_{j,t}}{P_t} Y_{j,t} - MC_t Y_{j,t} \nu_{j,t} \\ \Omega_{j,t} &= \frac{P_{j,t}}{P_t} Y_{j,t} - MC_t Y_{j,t} \nu_{j,t} \\ \Omega_t &= Y_t - MC_t a_j Y_t \left( \frac{P_{j,t}}{P_t} \right)^{-\sigma_2} \nu_{j,t} \end{aligned}$$

By definition of  $\nu_t$ :

$$\begin{aligned} Y_t \nu_t &= A_t K_t^\alpha L_t^{1-\alpha} \\ \Omega_t &= Y_t - Y_t MC_t \nu_t \end{aligned}$$

### B.1.3 Rewriting Price Evolution Equations Details

The evolution of the price dispersion equation for industry  $j$  (equation 62) can be rewritten as:

$$\begin{aligned} \nu_{j,t} &= \int_0^1 \left( \frac{P_{i,j,t}}{P_{j,t}} \right)^{-\sigma} di \\ &= \lambda_j \left( \frac{P_{j,t}^*}{P_{j,t}} \right)^{-\sigma} + (1 - \lambda_j) \int_0^1 \left( \frac{P_{i,j,t-1}}{P_{j,t}} \right)^{-\sigma} di \\ &= \lambda_j \left( \frac{P_{j,t}^*}{P_{j,t}} \right)^{-\sigma} + (1 - \lambda_j) \nu_{j,t-1} \Pi_{j,t}^\sigma \end{aligned}$$

The price aggregator for individual industries (equation 47) can be rewritten as:

$$\begin{aligned}
P_{j,t}^{1-\sigma} &= \int_0^1 P_{i,j,t}^{1-\sigma} di \\
&= \lambda_j P_{j,t}^{\star 1-\sigma} + (1-\lambda_j) P_{j,t-1}^{1-\sigma} \\
1 &= \lambda_j \left( \frac{P_{j,t}^{\star}}{P_{j,t}} \right)^{1-\sigma} + (1-\lambda_j) \Pi_{j,t}^{\sigma-1}
\end{aligned}$$

Also observe that by definition:

$$\Pi_{j,t} = \frac{P_{j,t}}{P_{j,t-1}} = \frac{P_{j,t}}{P_t} \frac{P_t}{P_{t-1}} \frac{P_{t-1}}{P_{j,t-1}}$$

And the final price evolution equation (equation 45) can be rewritten as:

$$\begin{aligned}
P_t^{1-\sigma_2} &= \sum_{j=1}^J a_j P_{j,t}^{1-\sigma_2} \\
1 &= \sum_{j=1}^J a_j \left( \frac{P_{j,t}}{P_t} \right)^{1-\sigma_2}
\end{aligned}$$

#### B.1.4 Firm Price Maximisation Details

The firm's problem is:

$$\max_{P_{j,t}^{\star}, Y_{i,j,t}} \mathbb{E}_t \left[ \sum_{k=0}^{\infty} \left( \frac{\beta_f}{\bar{R}} \right)^k (1-\lambda_j)^k \left[ \frac{P_{j,t}^{\star} Y_{i,j,t+k}}{P_{t+k}} - (1+\tau) MC_{j,t+k} Y_{i,j,t+k} \right] \right] \quad (99)$$

By equation 46:

$$Y_{i,j,t+k} = \left( \frac{P_{j,t}^{\star}}{P_{j,t+k}} \right)^{-\sigma} Y_{j,t+k} \quad (100)$$

Equation 100 can be input into equation 101 to get:

$$\max_{P_{j,t}^{\star}} \mathbb{E}_t \left[ \sum_{k=0}^{\infty} \left( \frac{\beta_f}{\bar{R}} \right)^k (1-\lambda_j)^k \left[ P_{j,t}^{\star 1-\sigma} P_{j,t+k}^{\sigma} Y_{j,t+k} - (1+\tau) MC_{j,t+k} P_{j,t}^{\star -\sigma} P_{j,t+k}^{\sigma} \frac{1}{P_{t+k}} Y_{j,t+k} \right] \right] \quad (101)$$

Taking FOCs:

$$\mathbb{E}_t \left[ \sum_{k=0}^{\infty} \left( \frac{\beta_f}{\bar{R}} \right)^k (1-\lambda_j)^k \left[ (1-\sigma) P_{j,t}^{\star -\sigma} P_{j,t+k}^{\sigma} \frac{1}{P_{t+k}} Y_{j,t+k} + \sigma (1+\tau) MC_{j,t+k} P_{j,t}^{\star -\sigma-1} P_{j,t+k}^{\sigma} Y_{j,t+k} \right] \right] \quad (102)$$

Rearranging:

$$\mathbb{E}_t \left[ \sum_{k=0}^{\infty} \left( \frac{\beta_f}{R} \right)^k (1 - \lambda_j)^k P_{j,t+k}^\sigma Y_{j,t+k} \left[ \frac{P_{j,t}^*}{P_{t+k}} - \frac{\sigma}{\sigma - 1} (1 + \tau) MC_{j,t+k} \right] \right] \quad (103)$$

Inputting  $Y_{j,t+k}$  and dividing by  $P_{j,t}^\sigma$ :

$$\mathbb{E}_t \left[ \sum_{k=0}^{\infty} \left( \frac{\beta_f}{R} \right)^k (1 - \lambda_j)^k \left( \frac{P_{j,t+k}}{P_{j,t}} \right)^\sigma \left( \frac{P_{j,t+k}}{P_{t+k}} \right)^{-\sigma_2} Y_{t+k} \left[ \frac{P_{j,t}^*}{P_{j,t}} \frac{P_{j,t}}{P_t} \frac{P_t}{P_{t+k}} - \frac{\sigma}{\sigma - 1} (1 + \tau) MC_{j,t+k} \right] \right] \quad (104)$$

Next, define:

$$U_{j,t} = \mathbb{E}_t \left[ \sum_{k=0}^{\infty} \left( \frac{\beta_f}{R} \right)^k (1 - \lambda_j)^k \left( \frac{P_{j,t+k}}{P_{j,t}} \right)^\sigma \left( \frac{P_{j,t+k}}{P_{t+k}} \right)^{-\sigma_2} Y_{t+k} \frac{P_t}{P_{t+k}} \right]$$

$$V_{j,t} = \mathbb{E}_t \left[ \sum_{k=0}^{\infty} \left( \frac{\beta_f}{R} \right)^k (1 - \lambda_j)^k \left( \frac{P_{j,t+k}}{P_{j,t}} \right)^\sigma \left( \frac{P_{j,t+k}}{P_{t+k}} \right)^{-\sigma_2} Y_{t+k} \frac{\sigma}{\sigma - 1} (1 + \tau) MC_{j,t+k} \right]$$

Then:

$$U_{j,t} \frac{P_{j,t}^*}{P_{j,t}} \frac{P_{j,t}}{P_t} - V_{j,t} = 0$$

$U_{j,t}, V_{j,t}$  can then be rewritten:

$$U_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\sigma_2} Y_t + \mathbb{E}_t \left[ \frac{\beta_f}{R} (1 - \lambda_j) \Pi_{j,t+1}^\sigma \Pi_{t+1}^{-1} U_{j,t+1} \right]$$

$$V_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\sigma_2} Y_t \frac{\sigma}{\sigma - 1} (1 + \tau) MC_t + \mathbb{E}_t \left[ \frac{\beta_f}{R} (1 - \lambda_j) \left( \frac{P_{j,t+1}}{P_{j,t}} \right)^\sigma V_{j,t+1} \right]$$

## C Steady State Details

In this section, I provide the fuller derivations of the steady state. The conditions are summarized in section 4.2

### C.1 Markup and Inflation Target

Firstly, note that in steady state:

$$\bar{\Pi} = \bar{\Pi}_j = \Pi^*$$

Equation 63 can be rewritten to get a steady state equation for  $\frac{P_j^*}{P_j}$  which is shown in equation 105. Equation 62 can be rewritten to get a steady state equation for  $\bar{v}_j$  which is shown in equation 106.

$$\overline{\left( \frac{P_j^*}{P_j} \right)} = \left( \frac{1 - \frac{1 - \lambda_j}{\Pi^{*1 - \sigma}}}{\lambda_j} \right)^{\frac{1}{1 - \sigma}} \quad (105)$$

$$\bar{v}_j = \frac{1}{1 - (1 - \lambda_j)\Pi^{*\sigma}} \lambda_j \left( \frac{P_j^*}{P_j} \right)^{-\sigma} \quad (106)$$

In steady state equations 66 to 68 become respectively:

$$\frac{\bar{P}_j}{\bar{P}} = \frac{\bar{V}_j}{\bar{U}_j} \left( \frac{\bar{P}_j^*}{\bar{P}_j} \right)^{-1} \quad (107)$$

$$\bar{U}_j = \frac{1}{1 - \frac{\beta_f}{R}(1 - \lambda_j)\bar{\Pi}^{\sigma-1}} \left( \frac{\bar{P}_j}{\bar{P}} \right)^{-\sigma_2} \bar{Y} \quad (108)$$

$$\bar{V}_j = \frac{1}{1 - \frac{\beta_f}{R}(1 - \lambda_j)\bar{\Pi}^\sigma} \left( \frac{\bar{P}_j}{\bar{P}} \right)^{-\sigma_2} \bar{Y} \frac{\sigma}{\sigma - 1} (1 + \tau) \bar{MC} \quad (109)$$

Equations 108 and 109 can be inputted into equation 107 to find equation 110.

$$\left( \frac{\bar{P}_j}{\bar{P}} \right) = \frac{\sigma}{\sigma - 1} \frac{1 - (1 - \lambda_j) \frac{\beta_f}{R} \Pi^{*\sigma-1}}{1 - (1 - \lambda_j) \frac{\beta_f}{R} \Pi^{*\sigma}} \left( \frac{\bar{P}_j^*}{\bar{P}_j} \right)^{-1} (1 + \tau) \bar{MC} \quad (110)$$

Equation 45 can be rewritten as equation 111.  $\left( \frac{\bar{P}_j}{\bar{P}} \right)$  can then be input from equation 110 into equation 111 to get equation 112.

$$\int_0^1 a_j \left( \frac{\bar{P}_j}{\bar{P}} \right)^{1-\sigma_2} dj = 1 \quad (111)$$

$$((1 + \tau) \bar{MC})^{1-\sigma_2} \int_0^1 a_j \left[ \frac{\sigma}{\sigma - 1} \frac{1 - (1 - \lambda_j) \frac{\beta_f}{R} \Pi^{*\sigma-1}}{1 - (1 - \lambda_j) \frac{\beta_f}{R} \Pi^{*\sigma}} \left( \frac{\bar{P}_j^*}{\bar{P}_j} \right)^{-1} \right]^{1-\sigma_2} dj = 1 \quad (112)$$

$\bar{MC}$  can be backed out from equation 112.  $\left( \frac{\bar{P}_j}{\bar{P}} \right)$  can then be found from equation 110.  $\bar{v}$  can be backed out by its definition (equation 55).  $\bar{m}$  can then be obtained by its definition (equation 57)

## C.2 Relative Asset Supply

The total assets supply available for the household to hold are capital and the value of firms given in equation 113.<sup>37</sup>

$$\bar{A}^s = (1 + n)\bar{K} + \bar{Z} \quad (113)$$

Applying equation 26 to equation 29 in the steady state yields a standard equation for the value of firms equation 114.

$$\bar{Z} = \frac{(1 + n)(\bar{\Omega} + \bar{Z})}{\bar{R}} \quad (114)$$

<sup>37</sup>  $\bar{K}$  needs to be multiplied by the population growth from one period to the next since assets are the assets that agents hold going forward to the next period.  $\bar{K}$  represents the per capita capital held at the start of a period. To have  $\bar{K}$  at the start of the next period, households must save  $(1 + n)\bar{K}$  at the end of the previous period.

$$\bar{Z} = \frac{\bar{\Omega}}{\frac{\bar{R}}{1+n} - 1} \quad (115)$$

Inputting equation 115 into equation 113 yields equation 116. Inputting equation 54 into equation 116 yields equation 117

$$\bar{A}^s = \frac{(\bar{R} - (1+n))\bar{K} + \bar{\Omega}}{\frac{\bar{R}}{1+n} - 1} \quad (116)$$

$$\bar{A}^s = \frac{\bar{Y} - (\delta + n)\bar{K} - \bar{W}\bar{L}}{\frac{\bar{R}}{1+n} - 1} \quad (117)$$

Equations 59 and 60 can be combined to find equation 118. Dividing equation 117 by labor income and inputting equations 60 and 118 yields equation 119.

$$\bar{W}\bar{L} = \frac{1-\alpha}{\alpha}(\bar{R} - 1 + \delta)\bar{K} \quad (118)$$

$$\bar{a}^s = \frac{\frac{m}{1-\alpha} - \frac{\alpha}{1-\alpha} \frac{\delta+n}{\bar{R}-1+\delta} - 1}{\frac{\bar{R}}{1+n} - 1} \quad (119)$$

### C.3 Relative Asset Demand

**Relative Labor** Applying arbitrage conditions on bonds and shares (equations 25 and 29) to equations 40 and 41 yields the result that assets held at the start of  $t + 1$  are the assets that were saved at the end of period  $t$  plus the return  $\bar{R}$ :

$$\bar{T} = \bar{R}\bar{S}^p$$

We can then apply this to equation 43 to yield:

$$\bar{C}_i + \bar{A}_{i+1} = \bar{W}\bar{L}_i + \bar{R}\bar{A}_i \quad (120)$$

where:

$$\bar{A}_i = \bar{s}_i^p \bar{T}$$

Next, iterate over equation 120 for a household from their first period of life to their last to get their intertemporal steady state budget constraint equation 121.

$$\sum_{i=0}^{M-1} \frac{\bar{C}_i}{\bar{R}^i} = \bar{W} \sum_{i=0}^{M-1} \frac{\bar{L}_i}{\bar{R}^i} \quad (121)$$

The Euler condition (equation 26) can be rewritten as equation 122.

$$\bar{C}_{i+1} = (\beta\bar{R})^{\frac{1}{\gamma}} \bar{C}_i \quad (122)$$

Iterating over equation 122 yields equation 123.

$$\bar{C}_i = (\beta \bar{R})^{\frac{i}{\gamma}} \bar{C}_0 \quad (123)$$

Inputting this back into equation 121 and simplifying yields equation 124.

$$\bar{C}_0 = \left( \sum_{i=0}^{M-1} \beta^{\frac{i}{\gamma}} \bar{R}^{\frac{i(1-\gamma)}{\gamma}} \right)^{-1} \bar{W} \sum_{i=0}^{M-1} \frac{\bar{L}_i}{\bar{R}^i} \quad (124)$$

Therefore, applying equation 122 to equation 124 yields an expression for consumption in any period in terms of  $\bar{R}$ :

$$\bar{C}_i = (\beta \bar{R})^{\frac{i}{\gamma}} \left( \sum_{i=0}^{M-1} \beta^{\frac{i}{\gamma}} \bar{R}^{\frac{i(1-\gamma)}{\gamma}} \right)^{-1} \bar{W} \sum_{i=0}^{M-1} \frac{\bar{L}_i}{\bar{R}^i} \quad (125)$$

Relative consumption for each cohort  $i$  is defined to be consumption by that cohort divided by labor income, as in equation 71:

$$\bar{c}_i = \frac{\bar{C}_i}{\bar{W} \bar{L}}$$

Thus, equation 125 can be rewritten as equation 126

$$\bar{c}_i = (\beta \bar{R})^{\frac{i}{\gamma}} \left( \sum_{i=0}^{M-1} \beta^{\frac{i}{\gamma}} \bar{R}^{\frac{i(1-\gamma)}{\gamma}} \right)^{-1} \sum_{i=0}^{M-1} \frac{\bar{L}_i}{\bar{R}^i} \quad (126)$$

**Endogenous Labor Relative Labor Supply** In the case with exogenous labor, relative consumption for each cohort has been rewritten purely in terms of  $\bar{R}$ . However, in the case with endogenous labor  $\frac{\bar{L}_i}{\bar{L}}$  are endogenous so the labor part of equation 126 (i.e.  $\sum_{i=0}^{M-1} \frac{\bar{L}_i}{\bar{L} \bar{R}^i}$ ) needs to be rewritten in terms of  $\bar{R}$  only.

Substituting the labor-leisure condition (equation 28) into the Euler condition (equation 26) yields the intertemporal labor supply condition:

$$\frac{v'(L_{i,t})}{W_t} = \beta R_{t+1} \frac{v'(L_{i,t+1})}{W_{t+1}} \quad (127)$$

Applying steady state and the disutility of working function to equation 127 yields:

$$x_i \bar{L}_i^\eta = \beta \bar{R} x_{i+1} \bar{L}_{i+1}^\eta \quad (128)$$

Rewriting equation 129:

$$\bar{L}_{i+1} = \left( \frac{1}{\beta \bar{R}} \right)^{\frac{1}{\eta}} \left( \frac{x_i}{x_{i+1}} \right)^{\frac{1}{\eta}} \bar{L}_i \quad (129)$$

Iterating over equation 129:

$$\bar{L}_i = \left( \frac{1}{\beta \bar{R}} \right)^{\frac{i}{\eta}} \left( \frac{x_0}{x_i} \right)^{\frac{i}{\eta}} \bar{L}_0 \quad (130)$$



Next, note that the (population weighted) total labor supply is given by:

$$\bar{L} = \left( \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \right)^{-1} \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \bar{L}_i \quad (131)$$

Inputting equation 130 into equation 131 yields:

$$\bar{L} = \left( \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \right)^{-1} \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \left( \frac{1}{\beta \bar{R}} \right)^{\frac{i}{\eta}} \left( \frac{x_0}{x_i} \right)^{\frac{i}{\eta}} \bar{L}_0 \quad (132)$$

Inputting  $\bar{L}_0$  from equation 132 into equation 130 yields the relative labor supplied by each cohort given by equation 133.

$$\frac{\bar{L}_i}{\bar{L}} = \left( \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \left( \frac{1}{\beta \bar{R}} \right)^{\frac{i}{\eta}} \left( \frac{x_0}{x_i} \right)^{\frac{i}{\eta}} \right)^{-1} \left( \frac{1}{\beta \bar{R}} \right)^{\frac{i}{\eta}} \left( \frac{x_0}{x_i} \right)^{\frac{i}{\eta}} \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \quad (133)$$

This has some economic intuition. When  $\bar{R}$  is higher, agents supply relatively less labor when they are old since they are already getting a high return on their savings so they don't need to work as much.

**Relative Asset Demand** Relative assets by cohort are defined in the same way as the definition of relative assets (equation 71):

$$\bar{a}_i = \frac{\bar{A}_i}{\bar{W}\bar{L}} \quad (134)$$

Equation 135 can be rewritten in terms of relative assets and relative consumption

$$\bar{c}_i + \bar{a}_{i+1} = \frac{\bar{L}_i}{\bar{L}} + \bar{R}\bar{a}_i \quad (135)$$

Note that  $\bar{a}_0 = \bar{a}_M = 0$  (since agents start with zero assets and have no need for assets when they are dead). Therefore, this yields  $M - 1$  equations from equation 135 and  $M - 1$  unknowns  $\bar{a}_1, \dots, \bar{a}_{M-1}$ . Thus,  $\bar{a}_i$  can be solved for by iterating over equation 135 starting from the beginning or end.

Total assets  $\bar{A}$  must equal the weighted sum of assets by cohort:

$$\bar{A} = \left( \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \right)^{-1} \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \bar{A}_i \quad (136)$$

Next, observe that the total relative asset demand is just given by the weighted sum of the relative assets held by each cohort. This can be shown by dividing equation 136 by labor income

and applying the definition of relative assets and relative assets by cohort (equations 71 and 134):

$$\bar{a}^d = \left( \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \right)^{-1} \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \bar{a}_i$$

Thus, it is possible to solve for  $\bar{a}^d$  using this process.

## D Generalized OLG Theory

In this section, results on the steady state of a generalized life cycle model with monopolistic firms are derived. To do this, the equilibrium of relative consumption is analyzed as opposed to the equilibrium of relative assets.

Define relative consumption as:

$$c = \frac{C}{WL}$$

By summing equation 126, it can be shown that the demand for relative consumption is given by:

$$\bar{c}^d = f(\bar{R})$$

where:

$$f(\bar{R}) = \left( \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \right)^{-1} \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} (\beta \bar{R})^{\frac{i}{\gamma}} \left( \sum_{i=0}^{M-1} \beta^{\frac{i}{\gamma}} \bar{R}^{\frac{i(1-\gamma)}{\gamma}} \right)^{-1} \sum_{i=0}^{M-1} \frac{\bar{L}_i}{\bar{L}} \frac{1}{\bar{R}^i} \quad (137)$$

By adjusting the resource condition, the following supply of relative consumption can be derived:

$$\bar{c}^s = \frac{\bar{m}}{(1-\alpha)} - \frac{\alpha(\delta+n)}{(\bar{R}-1+\delta)(1-\alpha)}$$

To begin, I show that the real interest rate must always be greater than the population growth.

**Theorem 1** ( $\bar{R} > 1+n$  Required when  $m > 1$ ). *When  $m > 1$ , require  $\bar{R} > 1+n$  for an equilibrium to hold.*

*Proof.* Firstly, note that when  $m > 1$ , this guarantees that  $\bar{\Omega} > 0$  since:

$$\bar{\Omega} = \bar{Y} \left( 1 - \frac{1}{m} \right)$$

The value of shares must be non-negative in equilibrium i.e.  $\bar{Z} \geq 0$ . By the arbitrage condition, it is also known that:

$$\bar{Z} \bar{R} = (1+n)(\bar{\Omega} + \bar{Z})$$

$$\bar{Z} = \frac{\bar{\Omega}}{\frac{\bar{R}}{1+n} - 1}$$

Observe that for  $\bar{Z} \geq 0$ , it must be the case that  $\bar{R} \geq 1 + n$ . □

Next, it is demonstrated that there must always exist an equilibrium.

**Theorem 2** (Existence of Equilibria). *When  $m = 1$ ,  $\bar{c}^d = \bar{c}^s$ .*

*When  $m > 1$ , there always exists an equilibrium when  $\bar{R} > 1 + n$ .*

*Proof.* Note the following results:

- When  $\bar{R} = 1 + n$ ,  $\bar{c}^d = 1$ .
- When  $m = 1$ ,  $\bar{R} = 1 + n$ ,  $\bar{c}^s = 1$ .
- When  $m > 1$ ,  $\bar{R} = 1 + n$ ,  $\bar{c}^s > 1$ .
- When  $\bar{R} \rightarrow \infty$ ,  $\bar{c}^d \rightarrow \infty$ .
- When  $\bar{R} \rightarrow \infty$ ,  $\bar{c}^s \rightarrow \frac{m}{1-\alpha}$ .

For an equilibrium, necessary condition  $\bar{c}^s = \bar{c}^d$  must be satisfied.

When  $m = 1$  (competitive firms): Observe that  $\bar{c}^s = \bar{c}^d$  when  $\bar{R} = 1$ . This does not automatically mean that there is a valid equilibrium at  $\bar{R} = 1$  since we have not proved that  $\bar{Z} \geq 0$  which is necessary for the equilibrium to seem realistic.

When  $m > 1$  (monopolistic firms): Observe that when  $\bar{R} = 1$ ,  $\bar{c}^d < \bar{c}^s$  but when  $\bar{R}$  is large,  $\bar{c}^d > \bar{c}^s$ . Therefore, since  $\bar{c}^d, \bar{c}^s$  are continuous functions of  $\bar{R}$ , there must be a point at which they cross and thus there exists some  $\bar{R} > 1 + n$  where  $\bar{c}^d = \bar{c}^s$ . □

Next, it is shown that an equilibrium must always be dynamically efficient.

**Theorem 3** ( $\bar{R} \geq 1 + n$  Guarantees Dynamic Efficiency). *When  $\bar{R} \geq 1 + n$ , there is dynamic efficiency. This implies that the equilibrium with monopolistic firms is always dynamically efficient.*

*Proof.*

$$\bar{C} = \bar{Y} - (\delta + n)\bar{K}$$

Inputting  $\bar{Y}$ :

$$\bar{C} = \frac{\bar{A}\bar{K}^\alpha \bar{L}^{1-\alpha}}{\bar{\nu}} - (\delta + n)\bar{K}$$

Differentiating:

$$\begin{aligned} \frac{d\bar{C}}{d\bar{K}} &= \frac{\alpha \bar{A}\bar{K}^{\alpha-1} \bar{L}^{1-\alpha}}{\bar{\nu}} - (\delta + n) \\ &= \frac{\bar{R} - 1 + \delta}{\bar{M}\bar{C}\bar{\nu}} - (\delta + n) \\ &= m(\bar{R} - 1 + \delta) - (\delta + n) \end{aligned}$$

This is greater than 0 when  $\bar{R} > 1 + n$ . □

## E Calibration Details

In this section, I provide more details on the calibration which was explained in section 4.5.

### E.1 Endogenous Labor Supply

Substituting the labor-leisure condition (equation 28) into the Euler condition (equation 26) yields the intertemporal labor supply condition:

$$\frac{v'(L_{i,t})}{W_t} = \beta R_{t+1} \frac{v'(L_{i,t+1})}{W_{t+1}} \quad (138)$$

In steady state this becomes:

$$x_i \bar{L}_i^\eta = \beta \bar{R} x_{i+1} \bar{L}_{i+1}^\eta \quad (139)$$

Equation 139 can be rewritten as:

$$x_{i+1} = \frac{1}{\beta \bar{R}} x_i \left( \frac{\bar{L}_i}{\bar{L}_{i+1}} \right)^\eta \quad (140)$$

Iterating over this yields:

$$x_i = \frac{1}{(\beta \bar{R})^i} x_0 \left( \frac{\bar{L}_0}{\bar{L}_i} \right)^\eta$$

Set  $x_0 = 1$ . To keep things simple, I just set  $x_i \forall i > 0$  so that the labor supply is the same as the exogenous case parameterization when  $\beta \bar{R} = 1$  so:

$$x_i = \left( \frac{\bar{L}_0}{\bar{L}_i} \right)^\eta$$

where the  $\bar{L}$  ratios are the same as in the exogenous labor case.

## F Results Details

In this section, I provide additional results relating to section 5 on what would happen if the central bank were to raise inflation from 2 to 4 percent.

The extent to which changing the degree of firm discounting i.e.  $\beta_f$  impacts the results is given in table 6.

Table 6: Impact of Changing Firm Discounting on the Policy Experiment Results

Firm Additional Discount ( $\beta_f$ )	0.89	0.935	0.972	1
Change in markup (p.p.)	-1.45	-1.07	-0.68	-0.32
Change in $\frac{C_{old}}{C_{young}}$ (%)	-7.34	-5.12	-3.10	-1.39
Change in K (%)	6.43	4.44	2.66	1.19
Change in r (p.p.)	-0.54	-0.38	-0.23	-0.10

The values of  $\beta_f$  in table 6 were picked to correspond to:  $\beta_f$  is the average discounting above the expected real rate that [Jagannathan et al. \(2016\)](#); [Graham and Harvey \(2011, 2012\)](#) actually find in their surveys;  $\beta_f = 0.935$  is the average discounting from WACC which is the value that is actually used;  $1/0.972 - 1$  was the average real rate paid by prime borrowers relative minus the average risk free rate between 1995 and 2007.

As the degree of discounting increases, the equilibrium real rate falls by more. This is because higher discounting means that firms care less about the future so lower the markup by more when inflation rises.

## G Idiosyncratic Labor Model

In this section, I present an alternative household setup where households have a similar life cycle structure but they face idiosyncratic labor shocks. This is an alternative parameterization that is discussed in section 5. To avoid complications, it is assumed there is no aggregate uncertainty. This assumption can be made since this model is only applied in the case of a long-run (steady state) equilibrium.

### G.1 Households

Household ages and the population are denoted in the same manner as section 3.1

Since there are idiosyncratic shocks within cohorts, it is necessary to consider how individuals within a cohort will respond. For each cohort, there is a continuum of individuals denoted  $h$  between 0 and 1 for each cohort  $i$ .

An individual  $h$  of cohort  $i$  at time  $t$  has a budget constraint given by equation 141. An agent either spends their money on consumption  $C_{h,i,t}$  or saves  $S_{h,i+1,t+1}$  for the next period. An agent receives direct income from working an exogenously set amount  $L_{h,i,t}$  at time  $t$  for real wage  $W$ . Savings from the previous period pay a gross return of  $R$ .

$$C_{h,i,t} + S_{h,i+1,t+1} \leq WL_{h,i,t} + RS_{h,i,t} \quad (141)$$

The amount that each individual works is dependent upon whether that household is em-

ployed or unemployed:

$$L_{h,i,t} = \begin{cases} L_{i,t} & \text{if employed} \\ U_{i,t} & \text{otherwise} \end{cases}$$

Whether or not the individual is employed is a Markov process.

Since there is no aggregate uncertainty, all assets must return the same. Thus, there is no need to specify exactly what assets agents hold for their savings. Instead, it can just be specified that a household  $h$  in cohort  $i$  at time  $t$  has savings  $S_{h,i,t}$  without specifying which assets they hold. Also, note that it is assumed that agents born today start with zero assets ( $S_{h,0,t} = 0$ ).

The agent has Epstein-Zin utility which allows the effects of risk aversion and income elasticity of substitution to be separated. This means their utility is defined recursively. Their relative risk aversion is denoted by  $\gamma$  and their intertemporal elasticity of substitution is denoted by  $\rho$ :

$$V_{i,t} = \left( (1 - \beta)C_{h,i,t}^{1-\rho} + \beta(\mathbb{E}_t[V_{i,t+1}^{1-\gamma}]^{\frac{1-\rho}{1-\gamma}}) \right)^{\frac{1}{1-\rho}} \quad (142)$$

Therefore, an agent of age  $k$  faces the following problem:

$$\max_{\{C_{h,i,t+i}, S_{h,i+1,t+i+1}\}_{i=k}^{M-1}} \mathbb{E}_t \left[ \sum_{i=0}^{M-k-1} \beta^{i-k} V_{i,t} \right] \quad (143)$$

s.t.  $\forall i \in k, \dots, M-1$ :

$$\begin{aligned} V_{i,t} &= \left( (1 - \beta)C_{h,i,t}^{1-\rho} + \beta(\mathbb{E}_t[V_{i,t+1}^{1-\gamma}]^{\frac{1-\rho}{1-\gamma}}) \right)^{\frac{1}{1-\rho}} \\ C_{h,i,t} + S_{h,i+1,t+1} &\leq WL_{h,i,t} + RS_{h,i,t} \\ S_{M,t+M} &\geq 0 \end{aligned} \quad (144)$$

As in the main model, relative assets are used. A value function approach is required due to the usage of Epstein Zin. This could be done using equations 143 and 144 but then savings will be a function of the wage which is determined on the supply side. This would require mean that it would be necessary to rerun the value function iteration each time the markup is changed. Instead, like in the non-idiosyncratic shock case, assets and consumption are written in relative terms so relative consumption and savings are defined as follows:

$$\begin{aligned} c_{h,i,t} &= \frac{C_{h,i,t}}{WL} \\ s_{h,i,t} &= \frac{S_{h,i,t}}{WL} \end{aligned}$$

The problem can then be rewritten as<sup>38</sup>:

$$\max_{\{C_{h,i,t+i}, S_{h,i+1,t+i+1}\}_{i=,}^{M-1}} \mathbb{E}_t \left[ \sum_{i=0}^{M-k-1} \beta^{i-k} V_{i,t} \right] \quad (145)$$

s.t.  $\forall i \in k, \dots, M-1$ :

$$\begin{aligned} V_{i,t} &= \left( (1-\beta)c_{h,i,t}^{1-\rho} + \beta(\mathbb{E}_t[V_{i,t+1}^{1-\gamma}]^{\frac{1-\rho}{1-\gamma}}) \right)^{\frac{1}{1-\rho}} \\ c_{h,i,t} + s_{h,i+1,t+1} &\leq \frac{L_{h,i,t}}{L} + R s_{h,i,t} \\ S_{M,t+M} &\geq 0 \end{aligned} \quad (146)$$

This then comes down to a series of value function problems. The value of savings and labor income in the final period of the agent's life is given by the utility of consuming all the remaining assets of the agent:

$$V_{i,M-1} = \left( (1-\beta) \left( \frac{L_{M-1}}{L} + R s_{M-1} \right)^{1-\rho} \right)^{\frac{1}{1-\rho}}$$

Then, working backwards, the value of an agent of cohort  $i$ 's savings and labor income can be computed using the value of an agent of cohort  $i+1$ 's savings and labor income.

Observe that  $s_i$  can be computed for every age given the value of  $R$ . Therefore, individual relative savings is effectively a function of  $R$ . Consequently, the aggregate relative savings is a function of the real interest rate and the population weighted sum of individual cohort labor supplies:

$$s(R) = \left( \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \right)^{-1} \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \int_0^1 s_{h,i}(R) dh$$

## G.2 Other Computations

I have shown the derivation for the demand for relative assets. The supply of relative assets and the relationship between the markup and inflation are the same as in appendix C

## H Empirics Robustness

In this section, I conduct robustness checks for the results presented in section 7. I discuss these results at the end of that section.

**OECD Members Pre-1975** The sample is reduced to consider only consider countries that were members of the OECD before 1975.

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<sup>38</sup>This has been simplified by cancelling a constant in the utility function.

Table 7: Inflation-Real OECD Original Members

$RealRate_{i,t}^{10yr}$	(1)	(2)	(3)	(4)
$Inflation_{i,t-4,t}$	-0.142** (0.047)	-0.169*** (0.048)	-0.546*** (0.072)	-0.909** (0.280)
country dummies		*	*	*
year dummies			*	*
econ controls				*
N	966	966	966	651

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.

Table 8: Inflation-Labor Share OECD Original Members

$LaborShare_{i,t}$	(1)	(2)	(3)	(4)
$Inflation_{i,t-4,t}$	0.269 (0.406)	0.517* (0.235)	0.164 (0.441)	0.351 (0.974)
country dummies		*	*	*
year dummies			*	*
econ controls				*
N	584	584	584	500

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.

**Low Inflation (< 10%)** The sample is reduced to consider only data points where long-run inflation exceeded 10%.

Table 9: Inflation-Real Low Inflation

$RealRate_{i,t}^{10yr}$	(1)	(2)	(3)	(4)
$Inflation_{i,t-4,t}$	-0.033 (0.063)	-0.059 (0.067)	-0.643*** (0.085)	-0.948*** (0.171)
country dummies		*	*	*
year dummies			*	*
econ controls				*
N	1088	1088	1088	808

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.



Table 10: Inflation-Labor Share Low Inflation

$LaborShare_{i,t}$	(1)	(2)	(3)	(4)
$Inflation_{i,t-4,t}$	-0.061 (0.512)	0.719** (0.242)	0.753* (0.312)	0.514 (0.416)
country dummies		*	*	*
year dummies			*	*
econ controls				*
N	745	745	745	678

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.

**Pre-2000** The sample is reduced to consider only before 2000.

Table 11: Inflation-Real Pre-2000

$RealRate_{i,t}^{10yr}$	(1)	(2)	(3)	(4)
$Inflation_{i,t-4,t}$	-0.323*** (0.042)	-0.354*** (0.034)	-0.426*** (0.049)	-0.451 (NaN)
country dummies		*	*	*
year dummies			*	*
econ controls				*
N	558	558	558	253

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.

Table 12: Inflation-Labor Share Pre-2000

$LaborShare_{i,t}$	(1)	(2)	(3)	(4)
$Inflation_{i,t-4,t}$	0.180* (0.080)	0.211*** (0.043)	0.200*** (0.048)	0.074 (NaN)
country dummies		*	*	*
year dummies			*	*
econ controls				*
N	286	286	286	198

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.

**Post-2000** The sample is reduced to consider only 2000 and after.

Table 13: Inflation-Real Post-2000

$RealRate_{i,t}^{10yr}$	(1)	(2)	(3)	(4)
$Inflation_{i,t-4,t}$	-0.228** (0.078)	-0.414*** (0.093)	-0.868*** (0.108)	-1.038*** (0.153)
country dummies		*	*	*
year dummies			*	*
econ controls				*
N	593	593	593	580

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.

Table 14: Inflation-Labor Share Post-2000

$LaborShare_{i,t}$	(1)	(2)	(3)	(4)
$Inflation_{i,t-4,t}$	-0.886 (0.804)	0.867*** (0.254)	1.049*** (0.282)	0.856* (0.339)
country dummies		*	*	*
year dummies			*	*
econ controls				*
N	527	527	527	523

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.

**Spurious Regression Checks** We consider differenced variables in order to check for spuriousness.

Table 15: Inflation-Real Spurious Regression Difference Check

$\Delta RealInterest_{i,t}^{10yr}$	(1)	(2)	(3)
$\Delta Inflation_{i,t-4,t}$	-0.611*** (0.057)	-0.702*** (0.079)	-0.850*** (0.100)
year dummies		*	*
econ controls			*
N	1116	1116	812

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.

Table 16: Inflation-Labor Share Spurious Regression Difference Check

$\Delta LaborShare_{i,t}$	(1)	(2)	(3)
$\Delta Inflation_{i,t-4,t}$	0.189*** (0.049)	0.187*** (0.049)	0.177*** (0.052)
year dummies		*	*
econ controls			*
N	781	781	707

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.

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